FACTORING TRINOMIALS OF THE FORM $ax^2 + bx + c$
BY GROUPING (the a • c Method)

Step 1: Look for a **GCF** and factor it out first.

Step 2: Multiply the coefficient of the leading term **a** by the constant term **c**. List the factors of this product (**a • c**) to find the pair of factors, f_1 and f_2, that sums to **b**, the coefficient of the middle term.

- When **c** is ***positive***, factors of **a • c** have the *same* sign –
  - If the middle term **bx** is *positive*, both factors are *positive*.
  - If the middle term **bx** is *negative*, both factors are *negative*.
  - Find the pair of factors that *adds* to **b**.
- When **c** is ***negative***, factors of **a • c** have *opposite* signs –
  - The *larger* of these factors has the *same sign* as the middle term.
  - Find the pair of factors that *subtracts* to **b**.

Step 3: Rewrite (split) the middle term **bx** using the factors, f_1 and f_2, found in Step 2. The expression now has 4 terms:

$$ax^2 + bx + c =$$
$$ax^2 + f_1x + f_2x + c$$

Step 4: Group the terms of the expression into binomial pairs as shown:

$$\left(ax^2 + f_1x\right) + \left(f_2x + c\right)$$

Step 5: Factor out a “gcf” from each pair. If the expression can be factored by grouping, the terms will share a common "binomial" factor.

Step 6: Factor out the common binomial factor to write the factorization.

Step 7: Check the result by multiplying.

EXAMPLE: Factor: $8x^3 - 34x^2 + 30x$

Step 1: Factor out the GCF **2x**:

$$8x^3 - 34x^2 + 30x = 2x\,(4x^2 - 17x + 15)$$

Step 2: Multiply **a • c**. Here, **a** = 4, **c** = 15, and **a • c** = 60. Next, list the factors of **60** to find the pair that *adds* to the middle term **–17x**. Note that the factors of 60 are both *negative*.

Factors of 60	Sum of Factors
−1 • −60	−61
−2 • −30	−32
−3 • −20	−23
−4 • −15	−19
−5 • −12	**−17**
−6 • −10	−16

Factors of **60** that add to **–17** are **−5** and **−12**.

Step 3: Rewrite the middle term **–17x** using the factors found in Step 2, **−5** and **−12**:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x\,(4x^2 - 17x + 15) \\ &= 2x\,(4x^2 - 5x - 12x + 15) \end{aligned}$$

Step 4: Group the terms into binomial pairs as shown:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x\,(4x^2 - 5x - 12x + 15) \\ &= 2x\,[(4x^2 - 5x) + (-12x + 15)] \end{aligned}$$

Step 5: Factor out a "gcf" from each group of terms. The gcf of the first group, $(4x^2 - 5x)$, is **x**; the gcf of the second group, $(-12x + 15)$, is **−3**:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x\,[(4x^2 - 5x) + (-12x + 15)] \\ &= 2x\,[\mathbf{x}\,(4x - 5)\ \mathbf{-3}\,(4x - 5)] \end{aligned}$$

The terms now share a common binomial factor: $(4x - 5)$.

Step 6: Factor out the common binomial factor **(4x – 5)** to write the factorization:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x\,[x\,\mathbf{(4x - 5)} - 3\,\mathbf{(4x - 5)}] \\ &= 2x\,(4x - 5)(x - 3) \end{aligned}$$

Step 7: Check the result by multiplying:

$$\begin{aligned} 2x\,(4x - 5)(x - 3) &= 2x\,(4x^2 - 12x - 5x + 15) \\ &= 2x\,(4x^2 - 17x + 15) \\ &= 8x^3 - 34x^2 + 30x \end{aligned}$$