

## RULES FOR EXPONENTS

Exponents are used to write repeated multiplication of the same factor. In the exponential expression  $2^3$ , the exponent 3 tells us how many times the base 2 is used as a factor:  $2^3 = 2 \cdot 2 \cdot 2$ . Similarly, in the expression  $x^4$ , the exponent tells us there are four factors of the base  $x$ :  $x^4 = x \cdot x \cdot x \cdot x$ .

When you simplify exponential expressions, it is important to understand the difference between the expressions  $-3^2$  and  $(-3)^2$ . The *lack* of parentheses in the expression  $-3^2$  means the exponent only raises the base "3" to the 2nd power. So  $-3^2$  evaluates as  $-(3 \cdot 3)$ . On the other hand, the *use* of parentheses in the expression  $(-3)^2$  means the exponent raises the base "-3" to the 2nd power. So  $(-3)^2$  evaluates as  $(-3) \cdot (-3)$ .

### 1. PRODUCT RULE: $a^m \cdot a^n = a^{m+n}$

To multiply exponential expressions with the same base, keep the base, add the exponents.

#### Examples:

$$1) \quad 2^3 \cdot 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$$

We get the same result when we add the powers:

$$2^3 \cdot 2^4 = 2^{3+4} = 2^7$$

$$2) \quad p \cdot p^6 \cdot p^8 = p^{1+6+8} = p^{15} \quad \leftarrow \text{When an exponent is not written, the power is "1".}$$

$$3) \quad (-3)^2 \cdot (-3)^6 = (-3)^{2+6} = (-3)^8 = 3^8 \quad \leftarrow \text{A negative number raised to an even power is positive.}$$

$$4) \quad -2x^4 (6x) (4x^3) = (-2 \cdot 6 \cdot 4) (x^{4+1+3}) = -48x^8 \quad \leftarrow \text{Use the associative property to multiply the coefficients and add the variable powers.}$$

### 2. QUOTIENT RULE: $\frac{a^m}{a^n} = a^{m-n}$

To divide exponential expressions with the same base, keep the base, subtract the exponents.

a) When the power in the numerator is greater than the power in the denominator (*top heavy*), the remainder goes in the numerator.

#### Examples:

$$1) \quad \frac{2^5}{2^2} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2}} = \frac{2^3}{1} = 2^3 = 8$$

*Notice*, we get the same result when we subtract the power in the denominator from the power in the numerator:

$$\frac{2^5}{2^2} = 2^{5-2} = 2^3 = 8$$

$$2) \frac{(-5)^7}{(-5)^4} = (-5)^{7-4} = (-5)^3 = -125 \quad \leftarrow \text{A negative number raised to an odd power is negative.}$$

$$3) \frac{x^{12}}{x^7} = x^{12-7} = x^5$$

$$4) \frac{9x^4y^5}{3x^2y} = \frac{\cancel{9}x^{4-2}y^{5-1}}{\cancel{3}} = \frac{3x^2y^4}{1} = 3x^2y^4$$

- b) When the power in the denominator is greater than the power in the numerator (*bottom heavy*), the remainder goes in the denominator and a "1" goes in the numerator when necessary.

**Examples:**

$$1) \frac{2^4}{2^6} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2} = \frac{1}{2^2} = \frac{1}{4}$$

We get the same result when we subtract the power in the numerator from the power in the denominator:

$$\frac{2^4}{2^6} = \frac{1}{2^{6-4}} = \frac{1}{2^2} = \frac{1}{4}$$

$$2) \frac{(-4)^3}{(-4)^8} = \frac{1}{(-4)^{8-3}} = \frac{1}{(-4)^5}$$

$$3) \frac{k^7}{k^{11}} = \frac{1}{k^{11-7}} = \frac{1}{k^4}$$

$$4) -\frac{7m^2n^5}{28m^8n^{10}} = -\frac{\cancel{7}}{\cancel{28}m^{8-2}n^{10-5}} = -\frac{1}{4m^6n^5}$$

$$5) \frac{36x^2y^3}{12x^4y} = \frac{\cancel{36}y^{3-1}}{\cancel{12}x^{4-2}} = \frac{3y^2}{1 \cdot x^2} = \frac{3y^2}{x^2} \quad \leftarrow \text{The x's are } \textit{bottom heavy}, \text{ the y's are } \textit{top heavy}.$$

3. **POWER RULE:**  $(a^m)^n = a^{m \cdot n}$

To raise an exponential expression to a power, keep the base, multiply the exponents.

**Examples:**

$$1) (2^4)^3 = 2^4 \cdot 2^4 \cdot 2^4 = 2^{4+4+4} = 2^{12}$$

We get the same result when we multiply the powers:

$$(2^4)^3 = 2^{4 \cdot 3} = 2^{12}$$

$$2) (-7^3)^5 = (-7)^{3 \cdot 5} = (-7)^{15} = -7^{15}$$

$$3) (x^2)^4 = x^{2 \cdot 4} = x^8$$

4. **POWER RULES FOR PRODUCTS AND QUOTIENTS**

a) **Power Rule for Products:**  $(ab)^n = a^n b^n$

To raise a product to a power, distribute the power to each base and multiply the exponents.

**Examples:**

$$1) (x^4y)^3 = x^{4 \cdot 3} y^{1 \cdot 3} = x^{12} y^3$$

$$2) (-2p^4qr^2)^4 = (-2)^{1 \cdot 4} p^{4 \cdot 4} q^{1 \cdot 4} r^{2 \cdot 4} = (-2)^4 p^{16} q^4 r^8 = 16p^{16} q^4 r^8$$

Think: "a negative raised to an even power is always positive."

b) **Power Rule for Quotients:**  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

To raise a quotient to a power, distribute the power to the numerator and the denominator and multiply the powers.

**Examples:**

$$1) \left(\frac{5}{x^2}\right)^4 = \frac{5^4}{x^8} = \frac{625}{x^8}$$

$$2) \left(\frac{-2x^2y}{z^3}\right)^2 = \frac{2^2 x^4 y^2}{z^6} = \frac{4x^4 y^2}{z^6}$$

## 5. EXPONENTS OF "1" AND "0"

### a) POWER OF "1:" $a^1 = a$

Any number or variable raised to the 1<sup>st</sup> power is the number or variable.

#### Examples:

$$1) 3^1 = 3$$

$$2) x^1 = x$$

$$3) (ab)^1 = a^1 b^1 = ab$$

### b) POWER OF "0:" $a^0 = 1$

Any number or variable raised to the zero power always equals 1.

When a number or variable is divided by itself, the result is 1. For example,

$$\frac{4}{4} = 1; \quad \frac{-3}{-3} = 1; \quad \frac{y}{y} = 1; \quad \frac{-2t}{-2t} = 1;$$

We get the same result when we apply the quotient rule:

$$\frac{2^5}{2^5} = 2^{5-5} = 2^0 = 1$$

#### Examples:

$$1) 5^0 = 1$$

$$2) x^0 = 1$$

$$3) -2y^0 = -2(1) = -2$$

$$4) -4^0 - 5^0 = -1 - 1 = -2$$

$$5) j^4 \cdot j^0 \cdot j^5 = j^{4+0+5} = j^9$$

## 6. RULE FOR NEGATIVE EXPONENTS: $a^{-n} = \frac{1}{a^n}$

A negative exponent means "take the reciprocal of." To simplify negative exponents, take the reciprocal of the base (flip) and make the power positive.

For example,  $2^{-3}$  is read "take the reciprocal of  $2^3$ ." Since  $2^3 = 8$ , its reciprocal is  $\frac{1}{2^3}$ , or  $\frac{1}{8}$ :

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

- a) If the power in the numerator is negative, move it to the denominator, and make the power positive.

**Examples:**

$$1) 6^{-3} = \frac{1}{6^3} = \frac{1}{216}$$

$$2) y^{-5} = \frac{1}{y^5}$$

$$3) \frac{x^{-2}}{y} = \frac{1}{x^2 y}$$

$$4) 2a^3 b^{-4} = \frac{2a^3 b^{-4}}{1} = \frac{2a^3}{b^4}$$

- b) If the power in the denominator is negative, move it to the numerator, and make the power positive.

**Examples:**

$$1) \frac{1}{5^{-2}} = 5^2 = 25$$

$$2) \frac{1}{p^{-7}} = p^7$$

$$3) \frac{x^5}{y^{-3}} = \frac{x^5 y^3}{1} = x^5 y^3$$

$$4) \frac{5}{m^{-3} n^{-2}} = \frac{5m^3 n^2}{1} = 5m^3 n^2$$

- c) If the numerator and the denominator have negative factors, simplify the negative powers first. Move negative powers in the numerator to the denominator; move negative powers in the denominator to the numerator.

**Examples:**

$$1) \frac{2^{-4}}{3^{-2}} = \frac{3^2}{2^4} = \frac{9}{16}$$

$$2) \frac{x^{-7}}{y^{-8}} = \frac{y^8}{x^7}$$

$$3) \frac{3^{-2} xy^{-3}}{x^{-4} y^3} = \frac{x \cdot x^4}{3^2 y^3 \cdot y^3} = \frac{x^5}{9y^6}$$

Simplify negative exponents first, and then apply the product and quotient rules as needed.

$$4) \left( \frac{5p^2}{q^3} \right)^{-2} = \left( \frac{q^3}{5p^2} \right)^2 = \frac{q^6}{5^2 p^4} = \frac{q^6}{25p^4}$$

Taking the reciprocal of the inside expression (“flipping it”) makes the outside power positive.

$$5) \frac{-2p^{-4}}{(p^3)^{-2}} = \frac{-2p^{-4}}{p^{-6}} = \frac{-2p^6}{p^4} = -2p^2 \leftarrow \text{Because } -2 \text{ is not raised to a power, the result is negative.}$$