A Selection of Resources for Success in this Mathematics Course

1.1 Study Skill Tips for Success in Mathematics
1.2 Symbols and Sets of Numbers
1.3 Fractions and Mixed Numbers
1.4 Exponents, Order of Operations, Variable Expressions, and Equations
1.5 Adding Real Numbers
1.6 Subtracting Real Numbers
1.7 Multiplying and Dividing Real Numbers
1.8 Properties of Real Numbers

✅ CHECK YOUR PROGRESS
Vocabulary Check
Chapter Highlights
Chapter Review
Getting Ready for the Test
Chapter Test

In this chapter, we review the basic symbols and words—the language—of arithmetic and introduce using variables in place of numbers. This is our starting place in the study of algebra.

For more information about the resources illustrated above, read Section 1.1.
1.1 Study Skill Tips for Success in Mathematics

Before reading Section 1.1, you might want to ask yourself a few questions.

1. When you took your last math course, were you organized? Were your notes and materials from that course easy to find, or were they disorganized and hard to find—if you saved them at all?

2. Were you satisfied—really satisfied—with your performance in that course? In other words, do you feel that your outcome represented your best effort?

If the answer is “no” to these questions, then it is time to make a change. Changing to or resuming good study skill habits is not a process you can start and stop as you please. It is something that you must remember and practice each and every day. To begin, continue reading this section.

**OBJECTIVE**

**1.** Getting Ready for This Course

Now that you have decided to take this course, remember that a *positive attitude* will make all the difference in the world. Your belief that you can succeed is just as important as your commitment to this course. Make sure you are ready for this course by having the time and positive attitude that it takes to succeed.

Make sure that you are familiar with the way that this course is being taught. Is it a traditional course, in which you have a printed textbook and meet with an instructor? Is it taught totally online, and your textbook is electronic and you e-mail your instructor? Or is your course structured somewhere in between these two methods? *(Not all of the tips that follow will apply to all forms of instruction.)*

Also make sure that you have scheduled your math course for a time that will give you the best chance for success. For example, if you are also working, you may want to check with your employer to make sure that your work hours will not conflict with your course schedule.

On the day of your first class period, double-check your schedule and allow yourself extra time to arrive on time in case of traffic problems or difficulty locating your classroom. Make sure that you are aware of and bring all necessary class materials.

**OBJECTIVE**

**2.** General Tips for Success

Below are some general tips that will increase your chance for success in a mathematics class. Many of these tips will also help you in other courses you may be taking.

- **Most important! Organize your class materials.** In the next couple pages, many ideas will be presented to help you organize your class materials—notes, any handouts, completed homework, previous tests, etc. In general, you MUST have these materials organized. All of them will be valuable references throughout your course and when studying for upcoming tests and the final exam. One way to make sure you can locate these materials when you need them is to use a three-ring binder. This binder should be used solely for your mathematics class and should be brought to each and every class or lab. This way, any material can be immediately inserted in a section of this binder and will be there when you need it.

- **Form study groups and/or exchange names and e-mail addresses.** Depending on how your course is taught, you may want to keep in contact with your fellow students. Some ways of doing this are to form a study group—whether in person or through the Internet. Also, you may want to ask if anyone is interested in exchanging e-mail addresses or any other form of contact.

- **Choose to attend all class periods.** If possible, sit near the front of the classroom. This way, you will see and hear the presentation better. It may also be easier for you to participate in classroom activities.

- **Do your homework.** You’ve probably heard the phrase “practice makes perfect” in relation to music and sports. It also applies to mathematics. You will find that the more time you spend solving mathematics exercises, the easier the process becomes. Be sure to schedule enough time to complete your assignments before the due date assigned by your instructor.

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Check your work. Review the steps you took while working a problem. Learn to check your answers in the original exercises. You may also compare your answers with the “Answers to Selected Exercises” section in the back of the book. If you have made a mistake, try to figure out what went wrong. Then correct your mistake. If you can’t find what went wrong, don’t erase your work or throw it away. Show your work to your instructor, a tutor in a math lab, or a classmate. It is easier for someone to find where you had trouble if he or she looks at your original work.

Learn from your mistakes and be patient with yourself. Everyone, even your instructor, makes mistakes. (That definitely includes me—Elayn Martin-Gay.) Use your errors to learn and to become a better math student. The key is finding and understanding your errors.

Was your mistake a careless one, or did you make it because you can’t read your own math writing? If so, try to work more slowly or write more neatly and make a conscious effort to carefully check your work.

Did you make a mistake because you don’t understand a concept? Take the time to review the concept or ask questions to better understand it.

Did you skip too many steps? Skipping steps or trying to do too many steps mentally may lead to preventable mistakes.

Know how to get help if you need it. It’s all right to ask for help. In fact, it’s a good idea to ask for help whenever there is something that you don’t understand. Make sure you know when your instructor has office hours and how to find his or her office. Find out whether math tutoring services are available on your campus. Check on the hours, location, and requirements of the tutoring service.

Don’t be afraid to ask questions. You are not the only person in class with questions. Other students are normally grateful that someone has spoken up.

Turn in assignments on time. This way, you can be sure that you will not lose points for being late. Show every step of a problem and be neat and organized. Also be sure that you understand which problems are assigned for homework. If allowed, you can always double-check the assignment with another student in your class.

MyMathLab® and MathXL®

Be aware of assignments and due dates set by your instructor. Don’t wait until the last minute to submit work online.

Objective

3. Knowing and Using Your Text

Flip through the pages of this text or view the e-text pages on a computer screen. Start noticing examples, exercise sets, end-of-chapter material, and so on. Every text is organized in some manner. Learn the way this text is organized by reading about and then finding an example in your text of each type of resource listed below. Finding and using these resources throughout your course will increase your chance of success.

• Practice Exercises. Each example in every section has a parallel Practice exercise. As you read a section, try each Practice exercise after you’ve finished the corresponding example. This “learn-by-doing” approach will help you grasp ideas before you move on to other concepts. Answers are at the back of the text.

• Symbols at the Beginning of an Exercise Set. If you need help with a particular section, the symbols listed at the beginning of each exercise set will remind you of the numerous resources available.

• Objectives. The main section of exercises in each exercise set is referenced by an example(s). There is also often a section of exercises entitled “Mixed Practice,” which is referenced by two or more examples or sections. These are mixed exercises written to prepare you for your next exam. Use all of this referencing if you have trouble completing an assignment from the exercise set.

• Icons (Symbols). Make sure that you understand the meaning of the icons that are beside many exercises. □ tells you that the corresponding exercise may be viewed on the video segment that corresponds to that section. ✓ tells you that this exercise is a writing exercise in which you should answer in complete sentences. △ tells you that the exercise involves geometry. □ tells you that this exercise is worked more efficiently with the aid of a calculator. Also, a feature called Graphing Calculator Explorations may be found before select exercise sets.
For use by Palm Beach State College only.

4  CHAPTER 1  Review of Real Numbers

- **Integrated Reviews.** Found in the middle of each chapter, these reviews offer you a chance to practice—in one place—the many concepts that you have learned separately over several sections.
- **End-of-Chapter Opportunities.** There are many opportunities at the end of each chapter to help you understand the concepts of the chapter.

**Vocabulary Checks** contain key vocabulary terms introduced in the chapter.

**Chapter Highlights** contain chapter summaries and examples.

**Chapter Reviews** contain review exercises. The first part is organized section by section and the second part contains a set of mixed exercises.

**Getting Ready for the Tests** contain conceptual exercises written to prepare students for chapter test directions as well as mixed sections of exercises.

**Chapter Tests** are sample tests to help you prepare for an exam. The Chapter Test Prep Videos found in the Interactive Lecture Series, MyMathLab, and YouTube provide the video solution to each question on each Chapter Test.

**Cumulative Reviews** start at Chapter 2 and are reviews consisting of material from the beginning of the book to the end of that particular chapter.

- **Student Resources in Your Textbook.** You will find a Student Resources section at the back of this textbook. It contains the following to help you study and prepare for tests:
  - **Study Skills Builders** contain study skills advice. To increase your chance for success in the course, read these study tips and answer the questions.
  - **Bigger Picture—Study Guide Outline** provides you with a study guide outline of the course, with examples.
  - **Practice Final** provides you with a Practice Final Exam to help you prepare for a final. The video solutions to each question are provided in the Interactive DVD Lecture Series and within MyMathLab®.

- **Resources to Check Your Work.** The Answers to Selected Exercises section provides answers to all odd-numbered section exercises and all integrated review and chapter test exercises.

**OBJECTIVE 4 Knowing and Using Video and Notebook Organizer Resources**

**Video Resources**

Below is a list of video resources that are all made by me—the author of your text, Elayn Martin-Gay. By making these videos, I can be sure that the methods presented are consistent with those in the text.

- **Interactive DVD Lecture Series.** Exercises marked with a are fully worked out by the author on the DVDs and within MyMathLab. The lecture series provides approximately 20 minutes of instruction per section and is organized by Objective.
- **Chapter Test Prep Videos.** These videos provide solutions to all of the Chapter Test exercises worked out by the author. They can be found in MyMathLab, the Interactive Lecture series, and YouTube. This supplement is very helpful before a test or exam.
- **Student Success Tips.** These video segments are about 3 minutes long and are daily reminders to help you continue practicing and maintaining good organizational and study habits.
- **Final Exam Videos.** These video segments provide solutions to each question. These videos can be found within MyMathLab and the Interactive Lecture Series.

**Notebook Organizer Resource**

This resource is in three-ring notebook ready form. It is to be inserted in a three-ring binder and completed. This resource is numbered according to the sections in your text to which they refer.

- **Video Organizer.** This organizer is closely tied to the Interactive Lecture (Video) Series. Each section should be completed while watching a lecture video on the same section. Once completed, you will have a set of notes to accompany the Lecture (Video) Series section by section.
If you have trouble completing assignments or understanding the mathematics, get help as soon as you need it! This tip is presented as an objective on its own because it is so important. In mathematics, usually the material presented in one section builds on your understanding of the previous section. This means that if you don’t understand the concepts covered during a class period, there is a good chance that you will not understand the concepts covered during the next class period. If this happens to you, get help as soon as you can.

Where can you get help? Many suggestions have been made in this section on where to get help, and now it is up to you to get it. Try your instructor, a tutoring center, or a math lab, or you may want to form a study group with fellow classmates. If you do decide to see your instructor or go to a tutoring center, make sure that you have a neat notebook and are ready with your questions.

1. Review your previous homework assignments.
2. Review any notes from class and section-level quizzes you have taken. (If this is a final exam, also review chapter tests you have taken.)
3. Review concepts and definitions by reading the Chapter Highlights at the end of each chapter.
4. Practice working out exercises by completing the Chapter Review found at the end of each chapter.
   (If this is a final exam, go through a Cumulative Review. There is one found at the end of the latest chapter that you have covered in your course.)
5. It is important that you place yourself in conditions similar to test conditions to find out how you will perform. In other words, as soon as you feel that you know the material, get a few blank sheets of paper and take a sample test. There is a Chapter Test available at the end of each chapter, or you can work selected problems from the Chapter Review. Your instructor may also provide you with a review sheet. During this sample test, do not use your notes or your textbook. Then check your sample test. If your sample test is the Chapter Test in the text, don’t forget that the video solutions are in MyMathLab, the Interactive Lecture Series, and YouTube. If you are not satisfied with the results, study the areas that you are weak in and try again.
6. On the day of the test, allow yourself plenty of time to arrive where you will be taking your exam.

When taking your test:
1. Read the directions on the test carefully.
2. Read each problem carefully as you take the test. Make sure that you answer the question asked.
3. Watch your time and pace yourself so that you can attempt each problem on your test.
4. If you have time, check your work and answers.
5. Do not turn your test in early. If you have extra time, spend it double-checking your work.
Managing Your Time

As a college student, you know the demands that classes, homework, work, and family place on your time. Some days you probably wonder how you’ll ever get everything done. One key to managing your time is developing a schedule. Here are some hints for making a schedule:

1. Make a list of all your weekly commitments for the term. Include classes, work, regular meetings, extracurricular activities, etc. You may also find it helpful to list such things as laundry, regular workouts, grocery shopping, etc.

2. Next, estimate the time needed for each item on the list. Also make a note of how often you will need to do each item. Don’t forget to include time estimates for the reading, studying, and homework you do outside of your classes. You may want to ask your instructor for help estimating the time needed.

3. In the exercise set that follows, you are asked to block out a typical week on the schedule grid given. Start with items with fixed time slots like classes and work.

4. Next, include the items on your list with flexible time slots. Think carefully about how best to schedule items such as study time.

5. Don’t fill up every time slot on the schedule. Remember that you need to allow time for eating, sleeping, and relaxing! You should also allow a little extra time in case some items take longer than planned.

6. If you find that your weekly schedule is too full for you to handle, you may need to make some changes in your workload, classload, or other areas of your life. You may want to talk to your advisor, manager or supervisor at work, or someone in your college’s academic counseling center for help with such decisions.

1. What is your instructor’s name?
2. What are your instructor’s office location and office hours?
3. What is the best way to contact your instructor?
4. Do you have the name and contact information of at least one other student in class?
5. Will your instructor allow you to use a calculator in this class?
6. Why is it important that you write step-by-step solutions to homework exercises and keep a hard copy of all work submitted?
7. Is there a tutoring service available on campus? If so, what are its hours? What services are available?
8. Have you attempted this course before? If so, write down ways that you might improve your chances of success during this next attempt.
9. List some steps that you can take if you begin having trouble understanding the material or completing an assignment. If you are completing your homework in MyMathLab® and MathXL®, list the resources you can use for help.
10. How many hours of studying does your instructor advise for each hour of instruction?
11. What does the ` icon in this text mean?
12. What does the △ icon in this text mean?
13. What does the ◀ icon in this text mean?
14. What are Practice exercises?
15. When might be the best time to work a Practice exercise?
16. Where are the answers to Practice exercises?
17. What answers are contained in this text and where are they?
18. What are Study Skills Builders and where are they?
19. What and where are Integrated Reviews?
20. How many times is it suggested that you work through the homework exercises in MathXL® before the submission deadline?
21. How far in advance of the assigned due date is it suggested that homework be submitted online? Why?
22. Chapter Highlights are found at the end of each chapter. Find the Chapter 1 Highlights and explain how you might use it and how it might be helpful.

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23. Chapter Reviews are found at the end of each chapter. Find the Chapter 1 Review and explain how you might use it and how it might be useful.

24. Chapter Tests are at the end of each chapter. Find the Chapter 1 Test and explain how you might use it and how it might be helpful when preparing for an exam on Chapter 1. Include how the Chapter Test Prep Videos may help. If you are working in MyMathLab® and MathXL®, how can you use previous homework assignments to study?

25. What is the Video Organizer? Explain the contents and how it might be used.

26. Explain how the Video Organizer can help you when watching a lecture video.

27. Read or reread Objective 7 and fill out the schedule grid below.

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### 1.2 Symbols and Sets of Numbers

#### OBJECTIVES

1. Use a Number Line to Order Numbers.
2. Translate Sentences into Mathematical Statements.
3. Identify Natural Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, and Real Numbers.
4. Find the Absolute Value of a Real Number.

#### OBJECTIVE 1 Using a Number Line to Order Numbers

We begin with a review of the set of natural numbers and the set of whole numbers and how we use symbols to compare these numbers. A set is a collection of objects, each of which is called a member or element of the set. A pair of brace symbols \{ \} encloses the list of elements and is translated as “the set of” or “the set containing.”

<table>
<thead>
<tr>
<th>Natural Numbers</th>
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<tbody>
<tr>
<td>The set of natural numbers is {1, 2, 3, 4, 5, 6, \ldots}.</td>
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</table>

<table>
<thead>
<tr>
<th>Whole Numbers</th>
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</thead>
<tbody>
<tr>
<td>The set of whole numbers is {0, 1, 2, 3, 4, \ldots}.</td>
</tr>
</tbody>
</table>

These numbers can be pictured on a number line. We will use number lines often to help us visualize distance and relationships between numbers.

To draw a number line, first draw a line. Choose a point on the line and label it 0. To the right of 0, label any other point 1. Being careful to use the same distance as from 0 to 1, mark off equally spaced distances. Label these points 2, 3, 4, 5, and so on. Since the whole numbers continue indefinitely, it is not possible to show every whole number on this number line. The arrow at the right end of the line indicates that the pattern continues indefinitely.

Picturing whole numbers on a number line helps us see the order of the numbers. Symbols can be used to describe concisely in writing the order that we see.

The equal symbol \( = \) means “is equal to.”

The symbol \( \neq \) means “is not equal to.”

These symbols may be used to form a mathematical statement. The statement might be true or it might be false. The two statements below are both true.

\[ 2 = 2 \] states that “two is equal to two.”

\[ 2 \neq 6 \] states that “two is not equal to six.”

If two numbers are not equal, one number is larger than the other.

The symbol \( > \) means “is greater than.”

The symbol \( < \) means “is less than.” For example,

\[ 3 < 5 \] states that “three is less than five.”

\[ 2 > 0 \] states that “two is greater than zero.”

On a number line, we see that a number to the right of another number is larger. Similarly, a number to the left of another number is smaller. For example, 3 is to the left of 5 on a number line, which means that 3 is less than 5, or \( 3 < 5 \). Similarly, 2 is to the right of 0 on a number line, which means 2 is greater than 0, or \( 2 > 0 \). Since 0 is to the left of 2, we can also say that 0 is less than 2, or \( 0 < 2 \).

The symbols \( \neq, <, \text{ and } > \) are called inequality symbols.
Section 1.2 Symbols and Sets of Numbers

Two other symbols are used to compare numbers.

The symbol \( \leq \) means “is less than or equal to.”

The symbol \( \geq \) means “is greater than or equal to.”

For example, \( 7 \leq 10 \) states that “seven is less than or equal to ten.”

This statement is true since \( 7 < 10 \) is true. If either \( 7 < 10 \) or \( 7 = 10 \) is true, then \( 7 \leq 10 \) is true.

\( 3 \geq 3 \) states that “three is greater than or equal to three.”

This statement is true since \( 3 = 3 \) is true. If either \( 3 > 3 \) or \( 3 = 3 \) is true, then \( 3 \geq 3 \) is true.

The symbols \( \leq \) and \( \geq \) are also called inequality symbols.

### Example 1

Insert \( <, >, \) or \( = \) in the space between each pair of numbers to make each statement true.

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<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>a.</td>
<td>2</td>
<td>3</td>
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<tr>
<td>b.</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>c.</td>
<td>72</td>
<td>27</td>
</tr>
</tbody>
</table>

**Solution**

a. \( 2 < 3 \) since 2 is to the left of 3 on a number line.

b. \( 7 > 4 \) since 7 is to the right of 4 on a number line.

c. \( 72 \geq 27 \) since 72 is to the right of 27 on a number line.

### Practice 1

Insert \( <, >, \) or \( = \) in the space between each pair of numbers to make each statement true.

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<tbody>
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<td>5</td>
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<td>b.</td>
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<td>4</td>
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<tr>
<td>c.</td>
<td>16</td>
<td>82</td>
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</table>

Two other symbols are used to compare numbers.

The symbol \( \leq \) means “is less than or equal to.”

The symbol \( \geq \) means “is greater than or equal to.”

For example, \( 7 \leq 10 \) states that “seven is less than or equal to ten.”

This statement is true since \( 7 < 10 \) is true. If either \( 7 < 10 \) or \( 7 = 10 \) is true, then \( 7 \leq 10 \) is true.

\( 3 \geq 3 \) states that “three is greater than or equal to three.”

This statement is true since \( 3 = 3 \) is true. If either \( 3 > 3 \) or \( 3 = 3 \) is true, then \( 3 \geq 3 \) is true.

The statement \( 6 \geq 10 \) is false since neither \( 6 > 10 \) nor \( 6 = 10 \) is true. The symbols \( \leq \) and \( \geq \) are also called inequality symbols.

### Example 2

Tell whether each statement is true or false.

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<td>a.</td>
<td>8 ( \geq ) 8</td>
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</tr>
</tbody>
</table>

**Solution**

a. True. Since \( 8 = 8 \) is true, then \( 8 \geq 8 \) is true.

b. True. Since \( 8 = 8 \) is true, then \( 8 \leq 8 \) is true.

c. False. Since neither \( 23 < 0 \) nor \( 23 = 0 \) is true, then \( 23 \leq 0 \) is false.

d. True. Since \( 23 > 0 \) is true, then \( 23 \geq 0 \) is true.

### Practice 2

Tell whether each statement is true or false.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>9 ( \geq ) 3</td>
<td>b.</td>
<td>3 ( \geq ) 8</td>
</tr>
</tbody>
</table>

### Objective 2

**Translating Sentences**

Now, let’s use the symbols discussed to translate sentences into mathematical statements.

### Example 3

Translate each sentence into a mathematical statement.

a. Nine is less than or equal to eleven.

b. Eight is greater than one.

c. Three is not equal to four.
Chapter 1

Review of Real Numbers

Solution

a. nine is less than or equal to eleven
   \[ 9 \leq 11 \]

b. eight is greater than one
   \[ 8 > 1 \]

c. three is not equal to four
   \[ 3 \neq 4 \]

Practice 3 Translate each sentence into a mathematical statement.

a. Three is less than eight.

b. Fifteen is greater than or equal to nine.

c. Six is not equal to seven.

Identifying Common Sets of Numbers

Whole numbers are not sufficient to describe many situations in the real world. For example, quantities less than zero must sometimes be represented, such as temperatures less than 0 degrees.

Numbers Less Than Zero on a Number Line

Numbers less than 0 are to the left of 0 and are labeled \(-1, -2, -3, \) and so on. A \(-\) sign, such as the one in \(-1,\) tells us that the number is to the left of 0 on a number line. In words, \(-1\) is read “negative one.” A \(+\) sign or no sign tells us that a number lies to the right of 0 on a number line. For example, \(3\) and \(+3\) both mean positive three.

The numbers we have pictured are called the set of integers. Integers to the left of 0 are called negative integers; integers to the right of 0 are called positive integers. The integer 0 is neither positive nor negative.

Example 4 Use an integer to express the number in the following. "Pole of Inaccessibility, Antarctica, is the coldest location in the world, with an average annual temperature of 72 degrees below zero." (Source: The Guinness Book of Records)

Solution The integer \(-72\) represents 72 degrees below zero.

Practice 4 Use an integer to express the number in the following. The elevation of Laguna Salada in Mexico is 10 meters below sea level. (Source: The World Almanac)
A problem with integers in real-life settings arises when quantities are smaller than some integer but greater than the next smallest integer. On a number line, these quantities may be visualized by points between integers. Some of these quantities between integers can be represented as a quotient of integers. For example,

- The point on a number line halfway between 0 and 1 can be represented by \( \frac{1}{2} \), a quotient of integers.
- The point on a number line halfway between 0 and \(-1\) can be represented by \( -\frac{1}{2} \).

Other quotients of integers and their graphs are shown to the left.

These numbers, each of which can be represented as a quotient of integers, are examples of rational numbers. It’s not possible to list the set of rational numbers using the notation that we have been using. For this reason, we will use a different notation.

\[
\text{Rational Numbers} = \left\{ \frac{a}{b} \middle| a \text{ and } b \text{ are integers and } b \neq 0 \right\}
\]

We read this set as “the set of all numbers \( \frac{a}{b} \) such that \( a \) and \( b \) are integers and \( b \) is not equal to 0.” Notice that every integer is also a rational number since each integer can be expressed as a quotient of integers. For example, the integer \( 5 \) is also a rational number since \( 5 = \frac{5}{1} \).

The number line also contains points that cannot be expressed as quotients of integers. These numbers are called irrational numbers because they cannot be represented by rational numbers. For example, \( \sqrt{2} \) and \( \pi \) are irrational numbers.

Both rational numbers and irrational numbers can be written as decimal numbers. The decimal equivalent of a rational number will either terminate or repeat in a pattern. For example, upon dividing we find that

\[
\begin{align*}
\text{Rational Numbers} & \quad \left\{ \frac{3}{4} = 0.75 \text{ (decimal number terminates or ends)} \right. \\
& \quad \left. \frac{2}{3} = 0.66666 \ldots \text{ (decimal number repeats in a pattern)} \right. 
\end{align*}
\]

The decimal representation of an irrational number will neither terminate nor repeat. For example, the decimal representations of irrational numbers \( \sqrt{2} \) and \( \pi \) are

\[
\begin{align*}
\text{Irrational Numbers} & \quad \left\{ \sqrt{2} = 1.414213562 \ldots \text{ (decimal number does not terminate or repeat in a pattern)} \right. \\
& \quad \left. \pi = 3.141592653 \ldots \text{ (decimal number does not terminate or repeat in a pattern)} \right. 
\end{align*}
\]

(For further review of decimals, see the Appendix.)

Combining the rational numbers with the irrational numbers gives the set of real numbers. One and only one point on a number line corresponds to each real number.

\[
\text{Real Numbers} = \left\{ \text{All numbers that correspond to points on a number line} \right\}
\]
Chapter 1
Review of Real Numbers

From our previous definitions, we have that every real number is either a rational number or an irrational number.

On the following number line, we see that real numbers can be positive, negative, or 0. Numbers to the left of 0 are called negative numbers; numbers to the right of 0 are called positive numbers. Positive and negative numbers are also called signed numbers.

Several different sets of numbers have been discussed in this section. The following diagram shows the relationships among these sets of real numbers.

**Example 5**
Given the set \( \left\{ -2, 0, \frac{1}{4}, -1.5, 112, -3, 11, \sqrt{2} \right\} \), list the numbers in this set that belong to the set of:

a. Natural numbers
b. Whole numbers
c. Integers
d. Rational numbers
e. Irrational numbers
f. Real numbers

**Solution**

a. The natural numbers are 11 and 112.
b. The whole numbers are 0, 11, and 112.
c. The integers are \(-3, -2, 0, 11, \) and 112.
d. Recall that integers are rational numbers also. The rational numbers are \(-3, -2, -1.5, 0, \frac{1}{4}, 11, \) and 112.
e. The irrational number is \( \sqrt{2} \).
f. The real numbers are all numbers in the given set.
We now extend the meaning and use of inequality symbols such as $<$ and $>$ to all real numbers.

**Practice**

Given the set $\left\{ 25, \frac{7}{3}, -15, -\frac{3}{4}, \sqrt{5}, -3.7, 8.8, -99 \right\}$, list the numbers in this set that belong to the set of:

- Natural numbers
- Whole numbers
- Integers
- Rational numbers
- Irrational numbers
- Real numbers

We now extend the meaning and use of inequality symbols such as $<$ and $>$ to all real numbers.

**Order Property for Real Numbers**

For any two real numbers $a$ and $b$, $a$ is less than $b$ if $a$ is to the left of $b$ on a number line.

**Example 6**

Insert $<, >$, or $=$ in the appropriate space to make each statement true.

- a. $-1 < 0$
- b. $7 = \frac{14}{2}$
- c. $-5 > -6$

**Solution**

- a. $-1 < 0$ since $-1$ is to the left of $0$ on a number line.
- b. $7 = \frac{14}{2}$ since $\frac{14}{2}$ simplifies to $7$.
- c. $-5 > -6$ since $-5$ is to the right of $-6$ on a number line.

**Practice**

Insert $<, >$, or $=$ in the appropriate space to make each statement true.

- a. $0 < 3$
- b. $15 > -5$
- c. $3 = \frac{12}{4}$

**Objective**

**Finding the Absolute Value of a Real Number**

A number line also helps us visualize the distance between numbers. The distance between a real number $a$ and $0$ is given a special name called the **absolute value** of $a$. “The absolute value of $a$” is written in symbols as $|a|$.

**Absolute Value**

The absolute value of a real number $a$, denoted by $|a|$, is the distance between $a$ and $0$ on a number line.

For example, $|3| = 3$ and $|-3| = 3$ since both $3$ and $-3$ are a distance of $3$ units from $0$ on a number line.
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Chapter 1

Review of Real Numbers

**Helpful Hint**

Since \(|a|\) is a distance, \(|a|\) is always either positive or 0, never negative. That is, for any real number \(a\), \(|a| \geq 0\).

**EXAMPLE 7** Find the absolute value of each number.

\[
\begin{array}{cccc}
\text{a. } \quad |4| & \quad \text{b. } \quad |-5| & \quad \text{c. } \quad |0| & \quad \text{d. } \quad \left| -\frac{1}{2} \right| & \quad \text{e. } \quad |5.6|
\end{array}
\]

**Solution**

\[
\begin{array}{cccc}
\text{a. } \quad |4| = 4 & \quad \text{b. } \quad |-5| = 5 & \quad \text{c. } \quad |0| = 0 & \quad \text{d. } \quad \left| -\frac{1}{2} \right| = \frac{1}{2} & \quad \text{e. } \quad |5.6| = 5.6
\end{array}
\]

**PRACTICE**

Find the absolute value of each number.

\[
\begin{array}{cccc}
\text{a. } \quad |-8| & \quad \text{b. } \quad |9| & \quad \text{c. } \quad |-2.5| & \quad \text{d. } \quad \frac{5}{11} & \quad \text{e. } \quad |\sqrt{3}|
\end{array}
\]

**EXAMPLE 8** Insert <, >, or = in the appropriate space to make each statement true.

\[
\begin{array}{cccccc}
\text{a. } \quad |0| & \quad \text{b. } \quad |-5| & \quad \text{c. } \quad |-3| & \quad \text{d. } \quad |5| & \quad \text{e. } \quad |-7|
\end{array}
\]

**Solution**

\[
\begin{array}{cccccc}
\text{a. } \quad |0| < 2 & \quad \text{b. } \quad |-5| = 5 & \quad \text{c. } \quad |-3| > |-2| & \quad \text{d. } \quad |5| < |6| & \quad \text{e. } \quad |-7| > |6|
\end{array}
\]

**PRACTICE** Insert <, >, or = in the appropriate space to make each statement true.

\[
\begin{array}{cccccc}
\text{a. } \quad |8| & \quad \text{b. } \quad |-8| & \quad \text{c. } \quad |-3| & \quad \text{d. } \quad |-11| & \quad \text{e. } \quad |3| & \quad |2|
\end{array}
\]

**Vocabulary, Readiness & Video Check**

Use the choices below to fill in each blank.

<table>
<thead>
<tr>
<th>real</th>
<th>natural</th>
<th>whole</th>
<th>inequality</th>
<th>integers</th>
<th>rational</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>b</td>
<td>)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. The _______ numbers are \{ 0, 1, 2, 3, 4, \ldots \}.
2. The _______ numbers are \{ 1, 2, 3, 4, 5, \ldots \}.
3. The symbols \(\neq, \leq,\) and \(>\) are called _______ symbols.
4. The _______ are \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}.
5. The _______ numbers are \{all numbers that correspond to points on a number line\}.
6. The _______ numbers are \(\left\{ \frac{a}{b} \mid a \text{ and } b \text{ are integers, } b \neq 0 \right\}\).
7. The _______ numbers are \{nonrational numbers that correspond to points on a number line\}.
8. The distance between a number \(b\) and 0 on a number line is _______.
1.2 Exercise Set

Insert <, >, or = in the appropriate space to make the statement true. See Example 1.

1. 7 3
2. 9 15
3. 6.26 6.26
4. 2.13 1.13
5. 0 7
6. 20 0
7. –2 2
8. –4 –6

9. The freezing point of water is 32°F. The boiling point of water is 212°F. Fahrenheit. Write an inequality statement using < or > comparing the numbers 32 and 212.

10. The freezing point of water is 0°C. The boiling point of water is 100°C. Write an inequality statement using < or > comparing the numbers 0 and 100.

11. An angle measuring 30° is shown and an angle measuring 45° is shown. Use the inequality symbol ≤ or ≥ to write a statement comparing the numbers 30 and 45.

12. The sum of the measures of the angles of a triangle is 180°. The sum of the measures of the angles of a parallelogram is 360°. Use the inequality symbol ≤ or ≥ to write a statement comparing the numbers 360 and 180.

Are the following statements true or false? See Examples 2 and 6.

13. 11 ≤ 11
14. 4 ≥ 7
15. 10 > 11
16. 17 > 16
17. 3 + 8 ≥ 3(8)
18. 8 · 8 ≤ 8 · 7
19. 9 > 0
20. 4 < 7
21. –6 > –2
22. 0 < –15

TRANSLATING

Write each sentence as a mathematical statement. See Example 3.

23. Eight is less than twelve.
24. Fifteen is greater than five.
25. Five is greater than or equal to four.
26. Negative ten is less than or equal to thirty-seven.
27. Fifteen is not equal to negative two.
28. Negative seven is not equal to seven.

Use integers to represent the values in each statement. See Example 4.

29. The highest elevation in California is Mt. Whitney, with an altitude of 14,494 feet. The lowest elevation in California is Death Valley, with an altitude of 282 feet below sea level. (Source: U.S. Geological Survey)
30. Driskill Mountain, in Louisiana, has an altitude of 535 feet. New Orleans, Louisiana, lies 8 feet below sea level. (Source: U.S. Geological Survey)
31. The number of graduate students at the University of Texas at Austin is 28,000 fewer than the number of undergraduate students. (Source: University of Texas at Austin)
32. The number of students admitted to the class of 2014 at UCLA is 80,784 fewer students than the number that had applied. (Source: UCLA)
33. Aaron Miller deposited $350 in his savings account. He later withdrew $126.
34. Aris Peña was deep-sea diving. During her dive, she ascended 30 feet and later descended 50 feet.

Tell which set or sets each number belongs to: natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. See Example 5.

35. 0
36. $\frac{1}{4}$
37. –2
38. $-\frac{1}{2}$

See Video 1.2

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.

9. In Example 2, why is the symbol < inserted between the two numbers?
10. Write the sentence given in Example 4 and translate it to a mathematical statement, using symbols.
11. Which sets of numbers does the number in Example 6 belong to?
12. Complete this statement based on the lecture given before Example 8. The _____ of a real number a, denoted by $|a|$, is the distance between a and 0 on a number line.

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16 CHAPTER 1 Review of Real Numbers

39. 6 40. 5
41. \(\frac{2}{3}\) 42. \(\sqrt{3}\)
43. \(-\sqrt{5}\) 44. \(-\frac{5}{9}\)

Tell whether each statement is true or false.
45. Every rational number is also an integer.
46. Every negative number is also a rational number.
47. Every natural number is positive.
48. Every rational number is also a real number.
49. 0 is a real number.
50. Every real number is also a rational number.
51. Every whole number is an integer.
52. \(\frac{1}{2}\) is an integer.
53. A number can be both rational and irrational.
54. Every whole number is positive.

Insert <, >, or = in the appropriate space to make a true statement. See Examples 6 through 8.
55. \(-10\) \(-100\) 56. \(-200\) \(-20\)
57. 32 5.2 58. 7.1 \(-7\)
59. \(\frac{18}{3}\) \(\frac{24}{3}\) 60. \(\frac{8}{2}\) \(\frac{12}{3}\)
61. \(-51\) \(-50\) 62. \(-20\) \(-200\)
63. \(-5\) \(-4\) 64. 0 \(\mid 0\)
65. \(-1\) \(\mid 1\) 66. \(\frac{2}{5}\) \(\frac{2}{5}\)
67. \(-2\) \(-3\) 68. \(-500\) \(-50\)
69. \(\mid 0\) \(-8\) 70. \(-12\) \(\frac{24}{2}\)

CONCEPT EXTENSIONS

The graph below is called a bar graph. This particular bar graph shows cranberry production from the top five cranberry-producing states.

Top Cranberry-Producing States (in millions of pounds)

Wisconsin 539
Oregon 40
Massachusetts 206
Washington 16
New Jersey 56

Millions of Pounds of Cranberries 2014

Data from National Agricultural Statistics Service

71. Write an inequality comparing the 2014 cranberry production in Oregon with the 2014 cranberry production in Washington.

72. Write an inequality comparing the 2014 cranberry production in Massachusetts with the 2014 cranberry production in Wisconsin.

73. Determine the difference between the 2014 cranberry production in Washington and the 2014 cranberry production in New Jersey.

74. According to the bar graph, which two states were the closest in terms of millions of pounds in 2014 cranberry crops?

This bar graph shows the number of people admitted into the Baseball Hall of Fame since its founding. Each bar represents a decade, and the height of the bar represents the number of Hall of Famers admitted in that decade.

Source: BASEBALL-Reference.com

75. In which decade(s) was the number of players admitted the greatest?
76. What was the greatest number of players admitted shown?
77. In which decade(s) was the number of players admitted greater than 40?
78. In which decade(s) was the number of players admitted fewer than 30?
80. Do you notice any trends shown by this bar graph?

The apparent magnitude of a star is the measure of its brightness as seen by someone on Earth. The smaller the apparent magnitude, the brighter the star. Use the apparent magnitudes in the table on page 17 to answer Exercises 81 through 86.

81. The apparent magnitude of the Sun is \(-26.7\). The apparent magnitude of the star Arcturus is \(-0.04\). Write an inequality statement comparing the numbers \(-0.04\) and \(-26.7\).
86. Which star listed is the dimmest?

Rewrite the following inequalities so that the inequality symbol points in the opposite direction and the resulting statement has the same meaning as the given one.

87. $25 \geq 20$

88. $-13 \leq 13$

89. $0 < 6$

90. $75 > 73$

91. $-10 > -12$

92. $-4 < -2$

93. In your own words, explain how to find the absolute value of a number.

94. Give an example of a real-life situation that can be described with integers but not with whole numbers.

<table>
<thead>
<tr>
<th>Star</th>
<th>Apparent Magnitude</th>
<th>Star</th>
<th>Apparent Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arcturus</td>
<td>-0.04</td>
<td>Spica</td>
<td>0.98</td>
</tr>
<tr>
<td>Sirius</td>
<td>-1.46</td>
<td>Rigel</td>
<td>0.12</td>
</tr>
<tr>
<td>Vega</td>
<td>0.03</td>
<td>Regulus</td>
<td>1.35</td>
</tr>
<tr>
<td>Antares</td>
<td>0.96</td>
<td>Canopus</td>
<td>-0.72</td>
</tr>
<tr>
<td>Sun</td>
<td>-26.7</td>
<td>Hadar</td>
<td>0.61</td>
</tr>
</tbody>
</table>


82. The apparent magnitude of Antares is 0.96. The apparent magnitude of Spica is 0.98. Write an inequality statement comparing the numbers 0.96 and 0.98.

83. Which is brighter, the Sun or Arcturus?

84. Which is dimmer, Antares or Spica?

85. Which star listed is the brightest?

1.3 Fractions and Mixed Numbers

**OBJECTIVES**

1. Write Fractions in Simplest Form.
2. Multiply and Divide Fractions.
3. Add and Subtract Fractions.

**OBJECTIVE 1 Writing Fractions in Simplest Form**

A quotient of two numbers such as $\frac{2}{9}$ is called a fraction. The parts of a fraction are:

- Fraction bar $\rightarrow \frac{2}{9} \leftarrow$ Numerator
- $\frac{2}{9} \leftarrow$ Denominator

A fraction may be used to refer to part of a whole. For example, $\frac{2}{9}$ of the circle above is shaded. The denominator 9 tells us how many equal parts the whole circle is divided into, and the numerator 2 tells us how many equal parts are shaded.

To simplify fractions, we can factor the numerator and the denominator. In the statement $3 \cdot 5 = 15$, 3 and 5 are called factors and 15 is the product. (The raised dot symbol indicates multiplication.)

$$3 \cdot 5 = 15$$

To factor 15 means to write it as a product. The number 15 can be factored as $3 \cdot 5$ or as $1 \cdot 15$.

A fraction is said to be simplified or in lowest terms when the numerator and the denominator have no factors in common other than 1. For example, the fraction $\frac{5}{11}$ is in lowest terms since 5 and 11 have no common factors other than 1.

To help us simplify fractions, we write the numerator and the denominator as products of prime numbers.
Chapter 1

Review of Real Numbers

Prime Number and Composite Number

A **prime number** is a natural number, other than 1, whose only factors are 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, and so on.

A natural number, other than 1, that is not a prime number is called a **composite number**.

Every composite number can be written as a product of prime numbers. We call this product of prime numbers the **prime factorization** of the composite number.

**EXAMPLE 1** Write each of the following numbers as a product of primes.

- **a.** 40
- **b.** 63

**Solution**

**a.** First, write 40 as the product of any two whole numbers other than 1.

\[ 40 = 4 \cdot 10 \]

Next, factor each of these numbers. Continue this process until all of the factors are prime numbers.

\[ 40 = 4 \cdot 10 \]
\[ = \frac{4}{4} \cdot \frac{10}{4} \]
\[ = 2 \cdot 2 \cdot 2 \cdot 5 \]

All the factors are now prime numbers. Then 40 written as a product of primes is

\[ 40 = 2 \cdot 2 \cdot 2 \cdot 5 \]

**b.** 63

\[ 63 = 9 \cdot 7 \]
\[ = \frac{9}{9} \cdot \frac{7}{9} \]
\[ = 3 \cdot 3 \cdot 7 \]

**PRACTICE** Write each of the following numbers as a product of primes.

- **a.** 36
- **b.** 200

To use prime factors to write a fraction in lowest terms (or simplified form), apply the fundamental principle of fractions.

**Fundamental Principle of Fractions**

If \( \frac{a}{b} \) is a fraction and \( c \) is a nonzero real number, then

\[ \frac{a \cdot c}{b \cdot c} = \frac{a}{b} \]

To understand why this is true, we use the fact that since \( c \) is not zero, \( \frac{c}{c} = 1 \).

\[ \frac{a \cdot c}{b \cdot c} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b} \]

We will call this process dividing out the common factor of \( c \).

**EXAMPLE 2** Simplify each fraction (write it in lowest terms).

- **a.** \( \frac{42}{49} \)
- **b.** \( \frac{11}{27} \)
- **c.** \( \frac{88}{20} \)
Section 1.3  Fractions and Mixed Numbers

Solution

a. Write the numerator and the denominator as products of primes; then apply the fundamental principle to the common factor 7.

\[
\frac{42}{49} = \frac{2 \cdot 3 \cdot 7}{7 \cdot 7} = \frac{2 \cdot 3 \cdot 7}{7 \cdot 7} = \frac{2 \cdot 3}{7} = \frac{6}{7}
\]

b. \( \frac{11}{27} = \frac{11}{3 \cdot 3 \cdot 3} \)

There are no common factors other than 1, so \( \frac{11}{27} \) is already in simplest form.

c. \( \frac{88}{20} = \frac{2 \cdot 2 \cdot 11}{2 \cdot 2 \cdot 5} = \frac{2 \cdot 2 \cdot 11}{2 \cdot 2 \cdot 5} = \frac{2 \cdot 11}{5} = \frac{22}{5} \)

Practice

2. Write each fraction in lowest terms.

a. \( \frac{63}{72} \)

b. \( \frac{64}{12} \)

c. \( \frac{7}{25} \)

Concept Check

Explain the error in the following steps.

a. \( \frac{15}{55} = \frac{1}{5} \)

b. \( \frac{6}{7} = \frac{5 + 1}{5 + 2} = \frac{1}{2} \)

Objective

2. Multiplying and Dividing Fractions

To multiply two fractions, multiply numerator times numerator to obtain the numerator of the product; multiply denominator times denominator to obtain the denominator of the product.

Multiplying Fractions

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad \text{if } b \neq 0 \text{ and } d \neq 0
\]

Example 3

Multiply \( \frac{2}{15} \) and \( \frac{5}{13} \). Simplify the product if possible.

Solution

\[
\frac{2}{15} \cdot \frac{5}{13} = \frac{2 \cdot 5}{15 \cdot 13} \quad \text{Multiply numerators.}
\]

\[
= \frac{2 \cdot 5}{3 \cdot 5 \cdot 13} \quad \text{Multiply denominators.}
\]

Next, simplify the product by dividing the numerator and the denominator by any common factors.

\[
= \frac{2 \cdot \frac{1}{3}}{5 \cdot 13}
\]

\[
= \frac{2}{39}
\]

Practice

3. Multiply \( \frac{3}{8} \) and \( \frac{7}{9} \). Simplify the product if possible.

Answers to Concept Check: answers may vary

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Before dividing fractions, we first define **reciprocals**. Two fractions are reciprocals of each other if their product is 1.

For example:
- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ because $\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1$.
- The reciprocal of 5 is $\frac{1}{5}$ because $5 \cdot \frac{1}{5} = \frac{5}{5} = \frac{5}{5} = 1$.

To divide fractions, multiply the first fraction by the reciprocal of the second fraction.

### Dividing Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}, \quad \text{if } b \neq 0, \ d \neq 0, \text{ and } c \neq 0$$

### EXAMPLE 4
Divide. Simplify all quotients if possible.

a. $\frac{4}{5} \div \frac{5}{16}$

**Solution**

The numerator and denominator have no common factors.

$$\frac{4}{5} \div \frac{5}{16} = \frac{4 \cdot 16}{5 \cdot 5} = \frac{64}{25}$$

b. $\frac{7}{10} \div 14$

$$\frac{7}{10} \div 14 = \frac{7}{10} \div \frac{14}{1} = \frac{7}{10} \cdot \frac{1}{2 \cdot 7} = \frac{1}{20}$$

c. $\frac{3}{8} \div \frac{3}{10}$

$$\frac{3}{8} \div \frac{3}{10} = \frac{3 \cdot 10}{8 \cdot 3} = \frac{1 \cdot 2 \cdot 5 \cdot 1}{2 \cdot 2 \cdot 5 \cdot 1} = \frac{5}{4}$$

### OBJECTIVE 3 Adding and Subtracting Fractions

To add or subtract fractions with the same denominator, combine numerators and place the sum or difference over the common denominator.

### Adding and Subtracting Fractions with the Same Denominator

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}, \quad \text{if } b \neq 0$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}, \quad \text{if } b \neq 0$$

### EXAMPLE 5
Add or subtract as indicated. Simplify each result if possible.

a. $\frac{2}{7} + \frac{4}{7}$

**Solution**

$$\frac{2}{7} + \frac{4}{7} = \frac{2 + 4}{7} = \frac{6}{7}$$

b. $\frac{3}{10} + \frac{2}{10}$

$$\frac{3}{10} + \frac{2}{10} = \frac{3 + 2}{10} = \frac{5}{10} = \frac{5}{2} = \frac{1}{2}$$

c. $\frac{9}{7} - \frac{2}{7}$

$$\frac{9}{7} - \frac{2}{7} = \frac{9 - 2}{7} = \frac{7}{7} = 1$$

d. $\frac{5}{3} - \frac{1}{3}$

$$\frac{5}{3} - \frac{1}{3} = \frac{5 - 1}{3} = \frac{4}{3}$$
To add or subtract with different denominators, we first write the fractions as equivalent fractions with the same denominator. We use the smallest or least common denominator, or LCD. (The LCD is the same as the least common multiple of the denominators.)

$$\frac{3}{4} \text{ and } \frac{12}{16} \text{ are equivalent fractions since they represent the same portion of a whole, as the diagram shows. Count the larger squares, and the shaded portion is } \frac{3}{4}. \text{ Count the smaller squares, and the shaded portion is } \frac{12}{16}. \text{ Thus, } \frac{3}{4} = \frac{12}{16}.$$

We can write equivalent fractions by multiplying a given fraction by 1, as shown in the next example. Multiplying a fraction by 1 does not change the value of the fraction.

**Example 6** Write $\frac{2}{5}$ as an equivalent fraction with a denominator of 20.

**Solution** Since $5 \cdot 4 = 20$, multiply the fraction by $\frac{4}{4}$. Multiplying by $\frac{4}{4} = 1$ does not change the value of the fraction.

$$\frac{2}{5} = \frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20}$$

Thus, $\frac{2}{5} = \frac{8}{20}$.

**Practice** Write $\frac{2}{3}$ as an equivalent fraction with a denominator of 21.

To add or subtract with different denominators, we first write the fractions as equivalent fractions with the same denominator. We use the smallest or least common denominator, or LCD. (The LCD is the same as the least common multiple of the denominators.)

**Example 7** Add or subtract as indicated. Write each answer in simplest form.

**Solution**

a. Fractions must have a common denominator before they can be added or subtracted. Since 20 is the smallest number that both 5 and 4 divide into evenly, 20 is the least common denominator (LCD). Write both fractions as equivalent fractions with denominators of 20. Since

$$\frac{2}{5} \cdot \frac{4}{4} = \frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20} \quad \text{and} \quad \frac{1}{4} \cdot \frac{5}{5} = \frac{1 \cdot 5}{4 \cdot 5} = \frac{5}{20}$$

then

$$\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$$
b. The LCD is 12. We write both fractions as equivalent fractions with denominators of 12.

\[
\frac{19}{6} - \frac{23}{12} = \frac{38}{12} - \frac{23}{12} = \frac{15}{12} = \frac{1 \cdot 5}{2 \cdot 2} = \frac{5}{4}
\]

c. The LCD for denominators 2, 22, and 11 is 22. First, write each fraction as an equivalent fraction with a denominator of 22. Then add or subtract from left to right.

\[
\frac{1}{2} = \frac{1}{2} \cdot \frac{11}{11} = \frac{11}{22}, \quad \frac{17}{22}, \quad \text{and} \quad \frac{2}{11} = \frac{2}{11} \cdot \frac{2}{2} = \frac{4}{22}
\]

Then

\[
\frac{1}{2} + \frac{17}{22} - \frac{2}{11} = \frac{11}{22} + \frac{17}{22} - \frac{4}{22} = \frac{24}{22} = \frac{12}{11}
\]

\[\square\]

**Practice 7** Add or subtract as indicated. Write answers in simplest form.

a. \(\frac{5}{11} + \frac{1}{7}\)  
b. \(\frac{5}{21} - \frac{1}{6}\)  
c. \(\frac{1}{3} + \frac{29}{30} - \frac{4}{5}\)

**Objective 4** Performing Operations on Mixed Numbers

To multiply or divide mixed numbers, first write each mixed number as an improper fraction. To recall how this is done, let’s write \(3\frac{1}{5}\) as an improper fraction.

\[3 \frac{1}{5} = 3 + \frac{1}{5} = \frac{15}{5} + \frac{1}{5} = \frac{16}{5}\]

Because of the steps above, notice that we can use a shortcut process for writing a mixed number as an improper fraction.

\[3 \frac{1}{5} = \frac{5 \cdot 3 + 1}{5} = \frac{16}{5}\]

**Example 8** Divide: \(2\frac{1}{8} + 1\frac{2}{3}\)

**Solution** First write each mixed number as an improper fraction.

\[
2 \frac{1}{8} = \frac{8 \cdot 2 + 1}{8} = \frac{17}{8}, \quad 1 \frac{2}{3} = \frac{3 \cdot 1 + 2}{3} = \frac{5}{3}
\]

Now divide as usual.

\[
2 \frac{1}{8} + 1 \frac{2}{3} = \frac{17}{8} + \frac{5}{3} = \frac{17 \cdot 3 + 5 \cdot 8}{24} = \frac{51}{40}
\]

The fraction \(\frac{51}{40}\) is improper. To write it as an equivalent mixed number, remember that the fraction bar means division and divide.

\[
\frac{11}{40}
\]

\[40)51\]

\[-40\]

\[11\]

Thus, the quotient is \(\frac{51}{40}\) or \(1 \frac{11}{40}\).

**Practice 8** Multiply: \(5\frac{1}{6} \cdot 4\frac{2}{5}\)

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As a general rule, if the original exercise contains mixed numbers, write the result as a mixed number if possible.

When adding or subtracting mixed numbers, you might want to use the following method.

**Example 9** Subtract: \( 50\frac{1}{6} - 38\frac{1}{3} \)

**Solution**

\[
50\frac{1}{6} = 50 + \frac{1}{6} = 49 + 1 + \frac{1}{6} = 49\frac{7}{6}
\]

\[-38\frac{1}{3} = -38 + \frac{2}{6} = -38\frac{2}{6} = -38\frac{2}{6} = 11\frac{5}{6}
\]

**Practice** Subtract: \( 76\frac{1}{12} - 35\frac{1}{4} \)

---

**Vocabulary, Readiness & Video Check**

Use the choices below to fill in each blank. Some choices may be used more than once.

simplified, reciprocals, equivalent, denominator, product, factors, fraction, numerator

1. A quotient of two numbers, such as \( \frac{5}{8} \), is called a(n) ____________.

2. In the fraction \( \frac{3}{11} \), the number 3 is called the ____________ and the number 11 is called the ____________.

3. To factor a number means to write it as a(n) ____________.

4. A fraction is said to be ____________ when the numerator and the denominator have no common factors other than 1.

5. In \( 7 \cdot 3 = 21 \), the numbers 7 and 3 are called ____________ and the number 21 is called the ____________.

6. The fractions \( \frac{2}{9} \) and \( \frac{9}{2} \) are called ____________.

7. Fractions that represent the same quantity are called ____________ fractions.

---

**Martin-Gay Interactive Videos**

Watch the section lecture video and answer the following questions.

8. What is the common factor in the numerator and denominator of Example 1? What principle is used to simplify this fraction?

9. During the solving of Example 3, what two things change in the first step?

10. What is the first step needed in order to subtract the fractions in Example 6 and why?

11. For Example 7, why is the sum not left as \( 4\frac{7}{6} \)?

---

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Chapter 1

Review of Real Numbers

1.3 Exercise Set MyMathLab

Represent the shaded part of each geometric figure by a fraction.

1.  

2.  

3.  

4.  

Write each number as a product of primes. See Example 1.

5. 33  

6. 60  

7. 98  

8. 27  

9. 20  

10. 56  

11. 75  

12. 32  

13. 45  

14. 24  

Write the fraction in lowest terms. See Example 2.

15. \(\frac{2}{4}\)  

16. \(\frac{3}{6}\)  

17. \(\frac{10}{15}\)  

18. \(\frac{15}{20}\)  

19. \(\frac{3}{7}\)  

20. \(\frac{5}{9}\)  

21. \(\frac{18}{30}\)  

22. \(\frac{42}{45}\)  

23. \(\frac{120}{244}\)  

24. \(\frac{360}{700}\)  

Multiply or divide as indicated. Simplify the answer if possible. See Examples 3 and 4.

25. \(\frac{1\frac{3}{2}}{4}\)  

26. \(\frac{1\frac{3}{11}}{5}\)  

27. \(\frac{2\frac{3}{3}}{4}\)  

28. \(\frac{7\frac{3}{8}}{21}\)  

29. \(\frac{1\frac{2}{7}}{12}\)  

30. \(\frac{7\frac{5}{10}}{21}\)  

31. \(\frac{3\frac{1}{20}}{4}\)  

32. \(\frac{3\frac{9}{10}}{5}\)  

33. \(\frac{7\frac{5}{10}}{21}\)  

34. \(\frac{3\frac{10}{63}}{25}\)  

35. \(\frac{1\frac{19}{3}}{4}\)  

The area of a plane figure is a measure of the amount of surface of the figure. Find the area of each figure below. (The area of a rectangle is the product of its length and width. The area of a triangle is \(\frac{1}{2}\) the product of its base and height.)

37.  

38.  

Add or subtract as indicated. Write the answer in lowest terms. See Example 5.

41. \(\frac{4\frac{1}{5}}{5}\)  

42. \(\frac{6\frac{1}{7}}{7}\)  

43. \(\frac{4\frac{1}{5}}{5}\)  

44. \(\frac{6\frac{1}{7}}{7}\)  

45. \(\frac{17\frac{10}{21}}{21}\)  

46. \(\frac{18\frac{11}{35}}{35}\)  

47. \(\frac{23\frac{4}{105}}{105}\)  

48. \(\frac{13\frac{35}{132}}{132}\)  

Write each fraction as an equivalent fraction with the given denominator. See Example 6.

49. \(\frac{7}{10}\) with a denominator of 30  

50. \(\frac{2}{3}\) with a denominator of 9  

51. \(\frac{2}{9}\) with a denominator of 18  

52. \(\frac{8}{7}\) with a denominator of 56  

53. \(\frac{4}{5}\) with a denominator of 20  

54. \(\frac{4}{5}\) with a denominator of 25  

Add or subtract as indicated. Write the answer in simplest form. See Example 7.

55. \(\frac{2\frac{3}{7}}{3}\)  

56. \(\frac{3\frac{1}{6}}{4}\)  

57. \(\frac{4\frac{1}{12}}{15}\)  

58. \(\frac{11\frac{1}{16}}{12}\)  

59. \(\frac{5\frac{5}{33}}{22}\)  

60. \(\frac{7\frac{8}{15}}{10}\)  

61. \(\frac{12\frac{5}{8}}{5}\)  

Each circle in Exercises 63–68 represents a whole, or 1. Use subtraction to determine the unknown part of the circle.

63.  

64.  

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The perimeter of a plane figure is the total distance around the figure. Find the perimeter of each figure in Exercises 97 and 98.

97. \[4 \text{ feet} + 10 \text{ feet} + 4 \text{ feet} + 15 \text{ feet} = 33 \text{ feet}\]

98. \[12 \text{ feet} + 2 \frac{3}{8} \text{ feet} + 9 \frac{1}{2} \text{ feet} + 16 \frac{1}{2} \text{ feet} = 40 \frac{5}{8} \text{ feet}\]

99. In your own words, explain how to add two fractions with different denominators.

100. In your own words, explain how to multiply two fractions.

The following trail chart is given to visitors at the Lakeview Forest Preserve.

<table>
<thead>
<tr>
<th>Trail Name</th>
<th>Distance (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robin Path</td>
<td>3 \frac{1}{2}</td>
</tr>
<tr>
<td>Red Falls</td>
<td>5 \frac{1}{2}</td>
</tr>
<tr>
<td>Green Way</td>
<td>2 \frac{1}{8}</td>
</tr>
<tr>
<td>Autumn Walk</td>
<td>1 \frac{3}{4}</td>
</tr>
</tbody>
</table>

101. How much longer is Red Falls Trail than Green Way Trail?

102. Find the total distance traveled by someone who hiked along all four trails.
The graph shown is called a circle graph or a pie chart. Use the graph to answer Exercises 103 through 106.

Fraction of U.S. Screens by Type

Analog 3/40
Digital but not 3D 11/20
Digital 3D 3/8

Data from Motion Picture Association of America

103. What fraction of U.S. movie screens are analog?

104. What fraction of U.S. movie screens are digital but not 3D?

105. What fraction of U.S. movie screens are digital?

106. What fraction of U.S. movie screens are analog or digital 3D?

For Exercises 107 through 110, determine whether the work is correct or incorrect. If incorrect, find the error and correct. See the Concept Check in this section.

107. \( \frac{12}{24} = \frac{2 + 6}{12} = \frac{1}{12} \)

108. \( \frac{30}{60} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 3 \cdot 5} = \frac{1}{2} \)

109. \( \frac{2}{7} + \frac{9}{7} = \frac{11}{14} \)

110. \( \frac{16}{28} = \frac{2 \cdot 5 + 6 \cdot 1}{2 \cdot 5 + 6 \cdot 3} = \frac{1}{3} \)

1.4 Exponents, Order of Operations, Variable Expressions, and Equations

OBJECTIVES

1. Define and Use Exponents and the Order of Operations.
2. Evaluate Algebraic Expressions, Given Replacement Values for Variables.
3. Determine Whether a Number Is a Solution of a Given Equation.
4. Translate Phrases into Expressions and Sentences into Statements.

OBJECTIVE 1 Using Exponents and the Order of Operations

Frequently in algebra, products occur that contain repeated multiplication of the same factor. For example, the volume of a cube whose sides each measure 2 centimeters is \(2 	imes 2 	imes 2\) cubic centimeters. We may use **exponential notation** to write such products in a more compact form. For example,

\[
2 \cdot 2 \cdot 2 \quad \text{may be written as} \quad 2^3.
\]

The 2 in \(2^3\) is called the **base**; it is the repeated factor. The 3 in \(2^3\) is called the **exponent** and is the number of times the base is used as a factor. The expression \(2^3\) is called an **exponential expression**.

\[
\text{base} \quad 2^3 \quad = \quad 2 \cdot 2 \cdot 2 \quad = \quad 8
\]

2 is a factor 3 times

\[
\text{Volume is} \quad (2 \cdot 2 \cdot 2) \quad \text{cubic centimeters.}
\]

**EXAMPLE 1** Evaluate the following.

a. \(3^2\) [read as “3 squared” or as “3 to the second power”]

b. \(5^3\) [read as “5 cubed” or as “5 to the third power”]

c. \(2^4\) [read as “2 to the fourth power”]

d. \(7^1\)

e. \(\left(\frac{3}{7}\right)^2\)

**Solution**

a. \(3^2 = 3 \cdot 3 = 9\)

b. \(5^3 = 5 \cdot 5 \cdot 5 = 125\)

c. \(2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16\)

d. \(7^1 = 7\)

e. \(\left(\frac{3}{7}\right)^2 = \frac{3 \cdot 3}{7 \cdot 7} = \frac{9}{49}\)
Using symbols for mathematical operations is a great convenience. However, the more operation symbols present in an expression, the more careful we must be when performing the indicated operation. For example, in the expression $2 + 3 \# 7$, do we add first or multiply first? To eliminate confusion, grouping symbols are used. Examples of grouping symbols are parentheses ( ), brackets [ ], braces { }, and the fraction bar. If we wish $2 + 3 \# 7$ to be simplified by adding first, we enclose $2 + 3$ in parentheses.

$2 + 3 \# 7 = 5 \# 7 = 35$

If we wish to multiply first, $3 \# 7$ may be enclosed in parentheses.

$2 + (3 \cdot 7) = 2 + 21 = 23$

To eliminate confusion when no grouping symbols are present, use the following agreed-upon order of operations.

**Order of Operations**

Simplify expressions using the order below. If grouping symbols such as parentheses are present, simplify expressions within those first, starting with the innermost set. If fraction bars are present, simplify the numerator and the denominator separately.

1. Evaluate exponential expressions.
2. Perform multiplications or divisions in order from left to right.
3. Perform additions or subtractions in order from left to right.

Now simplify $2 + 3 \cdot 7$. There are no grouping symbols and no exponents, so we multiply and then add.

$2 + 3 \cdot 7 = 2 + 21$

Multiply.

$= 23$

Add.

**Example 2**

Simplify each expression.

- $a. 6 + 3 + 5^2$
- $b. 20 \div 5 \cdot 4$
- $c. \frac{2(12 + 3)}{15} - 15$
- $d. 3 \cdot 4^2$
- $e. \frac{3}{2} + \frac{1}{2} - \frac{1}{2}$

**Solution**

- $a. 6 + 3 + 5^2$ first.

$6 + 3 + 5^2 = 6 + 3 + 25$

Next divide, then add.

$= 2 + 25$

Divide.

$= 27$

Add.

- $b. 20 \div 5 \cdot 4 = 4 \cdot 4$

$= 16$

**Helpful Hint**

Remember to multiply or divide in order from left to right.
c. First, simplify the numerator and the denominator separately.
\[
\frac{2(12 + 3)}{|-15|} = \frac{2(15)}{15} \quad \text{Simplify numerator and denominator separately.}
\]
\[
= \frac{30}{15} = 2 \quad \text{Simplify.}
\]
d. In this example, only the 4 is squared. The factor of 3 is not part of the base because no grouping symbol includes it as part of the base.
\[
3 \cdot 4^2 = 3 \cdot 16 \quad \text{Evaluate the exponential expression.}
\]
\[
= 48 \quad \text{Multiply.}
\]
e. The order of operations applies to operations with fractions in exactly the same way as it applies to operations with whole numbers.
\[
\frac{3}{2} - \frac{1}{2} = \frac{3}{4} - \frac{1}{2} \quad \text{Multiply.}
\]
\[
= \frac{3}{4} - \frac{2}{4} \quad \text{The least common denominator is 4.}
\]
\[
= \frac{1}{4} \quad \text{Subtract.}
\]

**Practice** 2. Simplify each expression.

a. \(6 + 3 \cdot 9\)  

b. \(4^3 + 8 + 3\)  

c. \(\left(\frac{2}{3}\right)^2 \cdot |-8|\)  

d. \(\frac{9(14 - 6)}{|-2|}\)  

e. \(\frac{7 \cdot 1}{4} - \frac{1}{4}\)

**Helpful Hint**

Be careful when evaluating an exponential expression. In \(3 \cdot 4^2\), the exponent 2 applies only to the base 4. In \((3 \cdot 4)^2\), the parentheses are a grouping symbol, so the exponent 2 applies to the product \(3 \cdot 4\). Thus, we multiply first.

\[
3 \cdot 4^2 = 3 \cdot 16 = 48 \quad (3 \cdot 4)^2 = (12)^2 = 144
\]

Expressions that include many grouping symbols can be confusing. When simplifying these expressions, keep in mind that grouping symbols separate the expression into distinct parts. Each is then simplified separately.

**Example 3**

Simplify: \(\frac{3 + |4 - 3| + 2^2}{6 - 3}\)

**Solution**
The fraction bar serves as a grouping symbol and separates the numerator and denominator. Simplify each separately. Also, the absolute value bars here serve as a grouping symbol. We begin in the numerator by simplifying within the absolute value bars.

\[
\frac{3 + |4 - 3| + 2^2}{6 - 3} = \frac{3 + |1| + 2^2}{6 - 3} \quad \text{Simplify the expression inside the absolute value bars.}
\]
\[
= \frac{3 + 1 + 2^2}{3} \quad \text{Find the absolute value and simplify the denominator.}
\]
\[
= \frac{3 + 1 + 4}{3} \quad \text{Evaluate the exponential expression.}
\]
\[
= \frac{8}{3} \quad \text{Simplify the numerator.}
\]

**Practice** 3. Simplify: \(\frac{6^2 - 5}{3 + |6 - 5| \cdot 8}\)

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Section 1.4  Exponents, Order of Operations, Variable Expressions, and Equations

**EXAMPLE 4**  Simplify: \(3[4 + 2(10 - 1)]\)

**Solution**  Notice that both parentheses and brackets are used as grouping symbols. Start with the innermost set of grouping symbols.

\[
3[4 + 2(10 - 1)] = 3[4 + 2(9)] \\
= 3[4 + 18] \\
= 3[22] \\
= 66
\]

Be sure to follow order of operations and resist the temptation to incorrectly add 4 and 2 first.

**PRACTICE**  Simplify: \(4[25 - 3(5 + 3)]\)

**EXAMPLE 5**  Simplify: \(\frac{8 + 2 \cdot 3}{2^2 - 1}\)

**Solution**

\[
\frac{8 + 2 \cdot 3}{2^2 - 1} = \frac{8 + 6}{4 - 1} = \frac{14}{3}
\]

**PRACTICE**  Simplify: \(\frac{36 \div 9 + 5}{5^2 - 3}\)

**OBJECTIVE**  Evaluating Algebraic Expressions

In algebra, we use symbols, usually letters such as \(x\), \(y\), or \(z\), to represent unknown numbers. A symbol that is used to represent a number is called a variable. An algebraic expression is a collection of numbers, variables, operation symbols, and grouping symbols. For example,

\[2x, \quad -3, \quad 2x + 10, \quad 5(p^2 + 1), \quad \text{and} \quad \frac{3y^2 - 6y + 1}{5}\]

are algebraic expressions.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x)</td>
<td>(2 \cdot x)</td>
</tr>
<tr>
<td>(5(p^2 + 1))</td>
<td>(5 \cdot (p^2 + 1))</td>
</tr>
<tr>
<td>(3y^2)</td>
<td>(3 \cdot y^2)</td>
</tr>
<tr>
<td>(xy)</td>
<td>(x \cdot y)</td>
</tr>
</tbody>
</table>

If we give a specific value to a variable, we can **evaluate an algebraic expression**. To evaluate an algebraic expression means to find its numerical value once we know the values of the variables.

Algebraic expressions are often used in problem solving. For example, the expression

\[16t^2\]

gives the distance in feet (neglecting air resistance) that an object will fall in \(t\) seconds.
EXAMPLE 6 Evaluate each expression if \(x = 3\) and \(y = 2\).

a. \(2x - y\) 
   \[
   2x - y = 2(3) - 2 \quad \text{Let } x = 3 \text{ and } y = 2. 
   
   = 6 - 2 \quad \text{Multiply.}
   
   = 4 \quad \text{Subtract.}
   
   \]

b. \(\frac{3x}{2y}\) 
   \[
   \frac{3x}{2y} = \frac{3 \cdot 3}{2 \cdot 2} = \frac{9}{4} \quad \text{Let } x = 3 \text{ and } y = 2.
   
   \]
c. \(\frac{x}{y} + \frac{y}{2}\) 
   \[
   \frac{x}{y} + \frac{y}{2} = \frac{3}{2} + \frac{2}{2} = \frac{5}{2}
   
   \]
d. \(x^2 - y^2\) 
   \[
   x^2 - y^2 = 3^2 - 2^2 = 9 - 4 = 5
   
   \]

PRACTICE 6 Evaluate each expression if \(x = 2\) and \(y = 5\).

a. \(2x + y\) 
   \[
   2x + y = 2(2) + 5 \quad \text{Let } x = 2 \text{ and } y = 5.
   
   = 4 + 5 \quad \text{Add.}
   
   = 9 \quad \text{Add.}
   
   \]

b. \(\frac{4x}{3y}\) 
   \[
   \frac{4x}{3y} = \frac{4 \cdot 2}{3 \cdot 5} = \frac{8}{15}
   
   \]
c. \(\frac{3}{x} + \frac{x}{y}\) 
   \[
   \frac{3}{x} + \frac{x}{y} = \frac{3}{2} + \frac{2}{5}
   
   \]
d. \(x^3 + y^2\) 

OBJECTIVE 3 Determining Whether a Number Is a Solution of an Equation

Many times, a problem-solving situation is modeled by an equation. An equation is a mathematical statement that two expressions have equal value. An equal sign “=” is used to equate the two expressions. For example,

\[3 + 2 = 5, \quad 7x = 35, \quad \frac{2(x - 1)}{3} = 0, \text{ and } I = PRT\] are all equations.

Helpful Hint

An equation contains an equal sign “=”. An algebraic expression does not.

CONCEPT CHECK

Which of the following are equations? Which are expressions?

a. \(5x = 8\)  
   b. \(5x - 8\)  
   c. \(12y + 3x\)  
   d. \(12y = 3x\)

When an equation contains a variable, deciding which values of the variable make the equation a true statement is called solving the equation for the variable. A solution of an equation is a value for the variable that makes the equation true. For example, 3 is a solution of the equation \(x + 4 = 7\) because if \(x\) is replaced with 3, the statement is true.

\[
\begin{align*}
  x + 4 &= 7 \\
  \downarrow \quad \text{Replace } x \text{ with } 3. \\
  3 + 4 &= 7 \quad \text{True}
\end{align*}
\]

Similarly, 1 is not a solution of the equation \(x + 4 = 7\) because \(1 + 4 = 7\) is not a true statement.

Answers to Concept Check:

equations: a, d; expressions: b, c.
EXAMPLE 7

Decide whether 2 is a solution of \(3x + 10 = 8x\).

Solution

Replace \(x\) with 2 and see if a true statement results.

\[
\begin{align*}
3x + 10 &= 8x & \text{Original equation} \\
3(2) + 10 &= 8(2) & \text{Replace } x \text{ with } 2. \\
6 + 10 &= 16 & \text{Simplify each side.} \\
16 &= 16 & \text{True}
\end{align*}
\]

Since we arrived at a true statement after replacing \(x\) with 2 and simplifying both sides of the equation, 2 is a solution of the equation.

PRACTICE 7

Decide whether 4 is a solution of \(9x - 6 = 7x\).

OBJECTIVE 4

Translating Phrases to Expressions and Sentences to Statements

Now that we know how to represent an unknown number by a variable, let’s practice translating phrases into algebraic expressions and sentences into statements. Oftentimes, solving problems requires the ability to translate word phrases and sentences into symbols. Below is a list of some key words and phrases to help us translate.

### Helpful Hint

Order matters when subtracting and dividing, so be especially careful with these translations.

<table>
<thead>
<tr>
<th>Addition (+)</th>
<th>Subtraction (−)</th>
<th>Multiplication (×)</th>
<th>Division (÷)</th>
<th>Equality (=)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum</td>
<td>Difference of</td>
<td>Product</td>
<td>Quotient</td>
<td>Equals</td>
</tr>
<tr>
<td>Plus</td>
<td>Minus</td>
<td>Times</td>
<td>Divide</td>
<td>Gives</td>
</tr>
<tr>
<td>Added to</td>
<td>Subtracted from</td>
<td>Multiply</td>
<td>Into</td>
<td>Is/Was/Should be</td>
</tr>
<tr>
<td>More than</td>
<td>Less than</td>
<td>Twice</td>
<td>Ratio</td>
<td>Yields</td>
</tr>
<tr>
<td>Increased by</td>
<td>Decreased by</td>
<td>Of</td>
<td>Divided by</td>
<td>Amounts to</td>
</tr>
<tr>
<td>Total</td>
<td>Less</td>
<td></td>
<td></td>
<td>Represents/Is the same as</td>
</tr>
</tbody>
</table>

EXAMPLE 8

Write an algebraic expression that represents each phrase. Let the variable \(x\) represent the unknown number.

a. The sum of a number and 3
b. The product of 3 and a number
c. Twice a number
d. 10 decreased by a number
e. 5 times a number, increased by 7

Solution

a. \(x + 3\) since “sum” means to add
b. \(3 \cdot x\) and \(3x\) are both ways to denote the product of 3 and \(x\)
c. \(2 \cdot x\) or \(2x\)
d. \(10 - x\) because “decreased by” means to subtract
e. \(\frac{5x}{5} + 7\) or \(5x + 7\) 

PRACTICE 8

Write an algebraic expression that represents each phrase. Let the variable \(x\) represent the unknown number.

a. Six times a number
b. A number decreased by 8
c. The product of a number and 9
d. Two times a number, plus 3
e. The sum of 7 and a number
Now let’s practice translating sentences into equations.

**EXAMPLE 9** Write each sentence as an equation or inequality. Let \( x \) represent the unknown number.

- **a.** The quotient of 15 and a number is 4.
- **b.** Three subtracted from 12 is a number.
- **c.** Four times a number, added to 17, is not equal to 21.
- **d.** Triple a number is less than 48.

**Solution**

- **a.** In words: the quotient of 15 and a number is 4
  
  \[ \frac{15}{x} = 4 \]

- **b.** In words: three subtracted from 12 is a number
  
  \[ 12 - 3 = x \]

  Care must be taken when the operation is subtraction. The expression \( 3 - 12 \) would be incorrect. Notice that \( 3 - 12 \neq 12 - 3 \).

- **c.** In words: four times a number added to 17 is not equal to 21
  
  \[ 4x + 17 \neq 21 \]

- **d.** In words: triple a number is less than 48
  
  \[ 3x < 48 \]

**PRACTICE 9** Write each sentence as an equation or inequality. Let \( x \) represent the unknown number.

- **a.** A number increased by 7 is equal to 13.
- **b.** Two less than a number is 11.
- **c.** Double a number, added to 9, is not equal to 25.
- **d.** Five times 11 is greater than or equal to an unknown number.
Exponents
To evaluate exponential expressions on a scientific calculator, find the key marked \( y^x \) or \( ^x \). To evaluate, for example, \( 3^5 \), press the following keys: \( 3 \ \boxed{y} \quad 5 \ \boxed{=} \) or \( 3 \ \boxed{^x} \quad 5 \ \boxed{=} \). The display should read \( 243 \) or \( 3^5 \quad 243 \).

Order of Operations
Some calculators follow the order of operations, and others do not. To see whether your calculator has the order of operations built in, use your calculator to find \( 2 + 3 \times 4 \). To do this, press the following sequence of keys:

\[ 2 + 3 \ \boxed{\times} \quad 4 \ \boxed{=} \]

The correct answer is 14 because the order of operations is to multiply before we add. If the calculator displays \( 14 \), then it has the order of operations built in.

Even if the order of operations is built in, parentheses must sometimes be inserted. For example, to simplify \( \frac{5}{12 - 7} \), press the keys

\[ 5 \ \boxed{-} \quad (1 \ \boxed{1} \ 2) \ \boxed{=} \]

The display should read \( 1 \) or \( \frac{5/(12 - 7)}{1} \).

Use a calculator to evaluate each expression.
1. \( 5^4 \)
2. \( 7^4 \)
3. \( 9^5 \)
4. \( 8^6 \)
5. \( 2(20 - 5) \)
6. \( 3(14 - 7) + 21 \)
7. \( 24(862 - 455) + 89 \)
8. \( 99 + (401 + 962) \)
9. \( \frac{4623 + 129}{36 - 34} \)
10. \( \frac{956 - 452}{89 - 86} \)

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank.

- equation
- variable
- base
- exponent
- expression
- solution
- solving
- grouping

1. In the expression \( 5^2 \), the 5 is called the _______ and the 2 is called the _______.
2. The symbols \( ( ) \), \[ \], and \{ \} are examples of _______ symbols.

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34  CHAPTER 1  Review of Real Numbers

3. A symbol that is used to represent a number is called a(n) ___________.
4. A collection of numbers, variables, operation symbols, and grouping symbols is called a(n) ___________.
5. A mathematical statement that two expressions are equal is called a(n) ___________.
6. A value for the variable that makes an equation a true statement is called a(n) ___________.
7. Deciding what values of a variable make an equation a true statement is called ___________ the equation.

Watch the section lecture video and answer the following questions.

8. In Example 3 and the lecture before, what is the main point made about the order of operations?
9. What happens with the replacement value for z in Example 6 and why?
10. Is the value 0 a solution of the equation given in Example 9? How is this determined?
11. Earlier in this video the point was made that equations have =, while expressions do not. In the lecture before Example 10, translating from English to math is discussed and another difference between expressions and equations is explained. What is it?

1.4  Exercise Set  MyMathLab®

Evaluate. See Example 1.

1. $3^5$
2. $2^5$
3. $3^3$
4. $4^3$
5. $1^5$
6. $1^8$
7. $5^1$
8. $8^1$
9. $7^2$
10. $9^2$
11. $\left(\frac{2}{3}\right)^4$
12. $\left(\frac{6}{11}\right)^2$
13. $\left(\frac{1}{5}\right)^3$
14. $\left(\frac{1}{2}\right)^5$
15. $(1.2)^2$
16. $(1.5)^2$
17. $(0.03)^4$

MIXED PRACTICE
Simplify each expression. See Examples 2 through 5.

19. $5 + 6 \cdot 2$
20. $8 + 5 \cdot 3$
21. $4 \cdot 8 - 6 \cdot 2$
22. $12 \cdot 5 - 3 \cdot 6$
23. $2(8 - 3)$
24. $5(6 - 2)$
25. $2 + (5 - 2) + 4^2$
26. $6 - 2 \cdot 2 + 2^2$
27. $5 \cdot 3^2$
28. $2 \cdot 5^2$
29. $\frac{1 \cdot 2}{3} - \frac{1}{6}$
30. $\frac{3 \cdot 1}{4} + \frac{2}{3}$

31. $2[5 + 2(8 - 3)]$
32. $3[4 + 3(6 - 4)]$
33. $\frac{19 - 3 \cdot 5}{6 - 4}$
34. $\frac{4 \cdot 3 + 2}{4 + 3 \cdot 2}$
35. $\frac{|6 - 2| + 3}{8 + 2 \cdot 5}$
36. $\frac{15 - |3 - 1|}{12 - 3 \cdot 2}$
37. $\frac{3 + 3(5 + 3)}{3^2 + 1}$
38. $\frac{3 + 6(8 - 5)}{4^2 + 2}$
39. $\frac{6 + |8 - 2| + 3^2}{18 - 3}$
40. $\frac{16 + |13 - 5| + 4^2}{17 - 5}$
41. $2 + 3[10(4 \cdot 5 - 16) - 30]$
42. $3 + 4[8(5 \cdot 5 - 20) - 39]$
43. $\left(\frac{2}{3}\right)^3 + \frac{1}{9} + \frac{1}{3}$
44. $\left(\frac{3}{8}\right)^2 + \frac{1}{4} + \frac{1}{8}$

For Exercises 45 and 46, match each expression in the first column with its value in the second column.

45. a. $(6 + 2) \cdot (5 + 3)$
   b. $(6 + 2) \cdot 5 + 3$
   c. $6 + 2 \cdot 5 + 3$
   d. $6 + 2 \cdot (5 + 3)$

46. a. $(1 + 4) \cdot 6 - 3$
   b. $1 + 4 \cdot (6 - 3)$
   c. $1 + 4 \cdot 6 - 3$
   d. $(1 + 4) \cdot (6 - 3)$
Evaluate each expression when \( x = 1, y = 3, \) and \( z = 5. \) See Example 6.

47. \( 3y \) 
48. \( 4x \) 
49. \( \frac{z}{5x} \) 
50. \( \frac{y}{2z} \) 
51. \( 3x - 2 \) 
52. \( 6y - 8 \) 
53. \( |2x + 3y| \) 
54. \( |5z - 2y| \) 
55. \( xy + z \) 
56. \( yz - x \) 
57. \( 5y^2 \) 
58. \( 2z^2 \)

Evaluate each expression if \( x = 12, y = 8, \) and \( z = 4. \) See Example 6.

59. \( \frac{x}{z} + 3y \) 
60. \( \frac{y}{z} + 8x \) 
61. \( x^2 - 3y + x \) 
62. \( y^2 - 3x + y \) 
63. \( \frac{x^2 + z}{y^2 + 2z} \) 
64. \( \frac{y^2 + x}{x^2 + 3y} \)

Neglecting air resistance, the expression \( 16t^2 \) gives the distance in feet an object will fall in \( t \) seconds.

65. Complete the chart below. To evaluate \( 16t^2, \) remember to first find \( t^2, \) then multiply by 16.

<table>
<thead>
<tr>
<th>Time ( t ) (in seconds)</th>
<th>Distance ( 16t^2 ) (in feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

66. Does an object fall the same distance during each second? Why or why not? (See Exercise 65.)

Decide whether the given number is a solution of the given equation. See Example 7.

67. Is 5 a solution of \( 3x + 30 = 9x? \)
68. Is 6 a solution of \( 2x + 7 = 3x? \)
69. Is 0 a solution of \( 2x + 6 = 5x - 1? \)
70. Is 2 a solution of \( 4x + 2 = x + 8? \)
71. Is 8 a solution of \( 2x - 5 = 5? \)
72. Is 6 a solution of \( 3x - 10 = 8? \)
73. Is 2 a solution of \( x + 6 = x + 6? \)
74. Is 10 a solution of \( x + 6 = x + 6? \)
75. Is 0 a solution of \( x = 5x + 15? \)
76. Is 1 a solution of \( 4 = 1 - x? \)

TRANSLATING
Write each phrase as an algebraic expression. Let \( x \) represent the unknown number. See Example 8.

77. Fifteen more than a number
78. A number increased by 9
79. Five subtracted from a number
80. Five decreased by a number
81. The ratio of a number and 4
82. The quotient of a number and 9
83. Three times a number, increased by 22
84. Twice a number, decreased by 72

TRANSLATING
Write each sentence as an equation or inequality. Use \( x \) to represent any unknown number. See Example 9.

85. One increased by two equals the quotient of nine and three.
86. Four subtracted from eight is equal to two squared.
87. Three is not equal to four divided by two.
88. The difference of sixteen and four is greater than ten.
89. The sum of 5 and a number is 20.
90. Seven subtracted from a number is 0.
91. The product of 7.6 and a number is 17.
92. 9.1 times a number equals 4.
93. Thirteen minus three times a number is 13.
94. Eight added to twice a number is 42.

CONCEPT EXTENSIONS
Fill in each blank with one of the following:
add subtract multiply divide

95. To simplify the expression \( 1 + 3 \cdot 6, \) first __________.
96. To simplify the expression \( (1 + 3) \cdot 6, \) first __________.
97. To simplify the expression \( (20 - 4) \cdot 2, \) first __________.
98. To simplify the expression \( 20 - 4 \div 2, \) first __________.
99. Are parentheses necessary in the expression \( 2 + (3 \cdot 5)? \) Explain your answer.
100. Are parentheses necessary in the expression \( (2 + 3) \cdot 5? \) Explain your answer.

Recall that perimeter measures the distance around a plane figure and area measures the amount of surface of a plane figure. The expression \( 2l + 2w \) gives the perimeter of the rectangle below (measured in units), and the expression \( lw \) gives its area (measured in square units). Complete the chart below for the given lengths and widths. Be sure to include units.

<table>
<thead>
<tr>
<th>Length: ( l )</th>
<th>Width: ( w )</th>
<th>Perimeter of Rectangle: ( 2l + 2w )</th>
<th>Area of Rectangle: ( lw )</th>
</tr>
</thead>
<tbody>
<tr>
<td>101. 4 in.</td>
<td>3 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>102. 6 in.</td>
<td>1 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>103. 5.3 in.</td>
<td>1.7 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>104. 4.6 in.</td>
<td>2.4 in.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
113. Write any expression, using 3 or more numbers, that simplifies to 11.

114. Write any expression, using 4 or more numbers, that simplifies to 7.

115. The area of a figure is the total enclosed surface of the figure. Area is measured in square units. The expression \( lw \) represents the area of a rectangle when \( l \) is its length and \( w \) is its width. Find the area of the following rectangular shaped lot.

```
120 feet
100 feet
```

116. A trapezoid is a four-sided figure with exactly one pair of parallel sides. The expression \( \frac{1}{2}h(B + b) \) represents its area, when \( B \) and \( b \) are the lengths of the two parallel sides and \( h \) is the height between these sides. Find the area if \( B = 15 \) inches, \( b = 7 \) inches, and \( h = 5 \) inches.

117. The expression \( \frac{d}{t} \) represents the average speed in miles per hour if a distance of \( d \) miles is traveled in \( t \) hours. Find the speed to the nearest whole number if the distance between Dallas, Texas, and Kaw City, Oklahoma, is 432 miles, and it takes Peter Callac 8.5 hours to drive the distance.

118. The expression \( \frac{I}{PT} \) represents the rate of interest being charged if a loan of \( P \) dollars for \( T \) years required \( I \) dollars in interest to be paid. Find the interest rate if a $650 loan for 3 years to buy a used IBM personal computer requires $126.75 in interest to be paid.

119. Why is \( 4^3 \) usually read as “four cubed”? (Hint: What is the volume of the cube below?)

120. Why is \( 8^2 \) usually read as “eight squared”? (Hint: What is the area of the square below?)

---

1.5 Adding Real Numbers

**OBJECTIVES**

1. Add Real Numbers.
2. Solve Applications That Involve Addition of Real Numbers.
3. Find the Opposite of a Number.

**Adding Real Numbers**

Real numbers can be added, subtracted, multiplied, divided, and raised to powers, just as whole numbers can. We use a number line to help picture the addition of real numbers. We begin by adding numbers with the same sign.

**Example**

Add: \( 3 + 2 \)

**Solution**

Recall that 3 and 2 are called addends. We start at 0 on a number line and draw an arrow representing the addend 3. This arrow is three units long and points to
Thinking of signed numbers as money earned or lost might help make addition more meaningful. Earnings can be thought of as positive numbers. If $1 is earned and later another $3 is earned, the total amount earned is $4. In other words, \(1 + 3 = 4\).

On the other hand, losses can be thought of as negative numbers. If $1 is lost and later another $3 is lost, a total of $4 is lost. In other words, \(-1 - 3 = -4\).

Using a number line each time we add two numbers can be time consuming. Instead, we can notice patterns in the previous examples and write rules for adding signed numbers. When adding two numbers with the same sign, notice that the sign of the sum is the same as the sign of the addends.

**Practice**

1. Add using a number line: \(2 + 4\)

**Example 2**

Add: \(-1 + (-2)\)

**Solution** Here, \(-1\) and \(-2\) are addends. We start at 0 on a number line and draw an arrow representing \(-1\). This arrow is one unit long and points to the left since \(-1\) is negative. From the tip of this arrow, we draw another arrow, representing \(-2\). The number below the tip of this arrow is the sum, \(-3\).

2. Add using a number line: \(-2 + (-3)\)

Thinking of signed numbers as money earned or lost might help make addition more meaningful. Earnings can be thought of as positive numbers. If $1 is earned and later another $3 is earned, the total amount earned is $4. In other words, \(1 + 3 = 4\).

On the other hand, losses can be thought of as negative numbers. If $1 is lost and later another $3 is lost, a total of $4 is lost. In other words, \((-1) + (-3) = -4\).

Using a number line each time we add two numbers can be time consuming. Instead, we can notice patterns in the previous examples and write rules for adding signed numbers. When adding two numbers with the same sign, notice that the sign of the sum is the same as the sign of the addends.

**Adding Two Numbers with the Same Sign**

Add their absolute values. Use their common sign as the sign of the sum.

**Example 3**

Add.

\(a. \ -3 + (-7) \quad b. \ -1 + (-20) \quad c. \ -2 + (-10)\)

**Solution** Notice that each time, we are adding numbers with the same sign.

\(a. \ -3 + (-7) = -10\) \quad Add their absolute values: \(3 + 7 = 10\).

Use their common sign.

\(b. \ -1 + (-20) = -21\) \quad Add their absolute values: \(1 + 20 = 21\).

Common sign.

\(c. \ -2 + (-10) = -12\) \quad Add their absolute values.

Common sign.
Adding numbers whose signs are not the same can also be pictured on a number line.

**EXAMPLE 4** Add: \(-4 + 6\)

**Solution**

Using temperature as an example, if a thermometer registers 4 degrees below 0 degrees and then rises 6 degrees, the new temperature is 2 degrees above 0 degrees. Thus, it is reasonable that \(-4 + 6 = 2\).

Once again, we can observe a pattern: when adding two numbers with different signs, the sign of the sum is the same as the sign of the addend whose absolute value is larger.

**Adding Two Numbers with Different Signs**

Subtract the smaller absolute value from the larger absolute value. Use the sign of the number whose absolute value is larger as the sign of the sum.

**EXAMPLE 5** Add.

\(a.\) \(3 + (-7)\)  \(b.\) \(-2 + 10\)  \(c.\) \(0.2 + (-0.5)\)

**Solution** Notice that each time, we are adding numbers with different signs.

\(a.\) \(3 + (-7) = -4 \leftarrow\) Subtract their absolute values: \(7 - 3 = 4\). The negative number, \(-7\), has the larger absolute value so the sum is negative.

\(b.\) \(-2 + 10 = 8 \leftarrow\) Subtract their absolute values: \(10 - 2 = 8\). The positive number, 10, has the larger absolute value so the sum is positive.

\(c.\) \(0.2 + (-0.5) = -0.3 \leftarrow\) Subtract their absolute values: \(0.5 - 0.2 = 0.3\). The negative number, \(-0.5\), has the larger absolute value so the sum is negative.

**PRACTICE** Add.

\(a.\) \(15 + (-18)\)  \(b.\) \(-19 + 20\)  \(c.\) \(-0.6 + 0.4\)

In general, we have the following:

**Adding Real Numbers**

To add two real numbers

1. with the same sign, add their absolute values. Use their common sign as the sign of the answer.
2. with different signs, subtract their absolute values. Give the answer the same sign as the number with the larger absolute value.


**Example 6**  
Add.

a. \(-8 + (-11)\)  
b. \(-5 + 35\)  
c. \(0.6 + (-1.1)\)  
d. \(\frac{7}{10} + \left(-\frac{1}{10}\right)\)  
e. \(11.4 + (-4.7)\)  
f. \(-\frac{3}{8} + \frac{2}{5}\)

**Solution**

a. \(-8 + (-11) = -19\)  
   Same sign. Add absolute values and use the common sign.

b. \(-5 + 35 = 30\)  
   Different signs. Subtract absolute values and use the sign of the number with the larger absolute value.

c. \(0.6 + (-1.1) = -0.5\)  
   Different signs.

d. \(\frac{7}{10} + \left(-\frac{1}{10}\right) = \frac{8}{10} = \frac{4}{5}\)  
   Same sign.

e. \(11.4 + (-4.7) = 6.7\)

f. \(-\frac{3}{8} + \frac{2}{5} = \frac{15}{40} + \frac{16}{40} = \frac{1}{40}\)

**Practice 6**  
Add.

a. \(\frac{3}{5} + \left(-\frac{2}{5}\right)\)  
b. \(3 + (-9)\)  
c. \(2.2 + (-1.7)\)  
d. \(-\frac{2}{7} + \frac{3}{10}\)

**Example 7**  
Add.

a. \(3 + (-7) + (-8)\)  
b. \([7 + (-10)] + [-2 + |-4|]\)

**Solution**

a. Perform the additions from left to right.  
   \(3 + (-7) + (-8) = -4 + (-8) = -12\)  
   Adding numbers with different signs.  
   Adding numbers with like signs.

b. Simplify inside brackets first.  
   Add.

**Practice 7**  
Add.

a. \(8 + (-5) + (-9)\)  
b. \([-8 + 5] + [-5 + |-2|]\)

**Concept Check**  
What is wrong with the following calculation?

\(5 + (-22) = 17\)

**Answer to Concept Check:**  
\(5 + (-22) = -17\)

**Objective 2**  
Solving Applications by Adding Real Numbers  
Positive and negative numbers are often used in everyday life. Stock market returns show gains and losses as positive and negative numbers. Temperatures in cold climates...
often dip into the negative range, commonly referred to as “below zero” temperatures. Bank statements report deposits and withdrawals as positive and negative numbers.

**Example 8** Calculating Temperature

In Philadelphia, Pennsylvania, the record extreme high temperature is 104°F. Decrease this temperature by 115 degrees, and the result is the record extreme low temperature. Find this temperature. *(Source: National Climatic Data Center)*

**Solution:**

\[
\text{extreme low temperature} = \text{extreme high temperature} + \text{decrease of 115°}
\]

\[
= 104 + (-115)
\]

\[= -11\]

The record extreme low temperature in Philadelphia, Pennsylvania, is \(-11\)°F.

**Practice 8** If the temperature was \(-7\)° Fahrenheit at 6 a.m., and it rose 4 degrees by 7 a.m. and then rose another 7 degrees in the hour from 7 a.m. to 8 a.m., what was the temperature at 8 a.m.?

**Objective 3** Finding the Opposite of a Number

To help us subtract real numbers in the next section, we first review the concept of opposites. The graphs of 4 and \(-4\) are shown on a number line below.

Notice that 4 and \(-4\) lie on opposite sides of 0, and each is 4 units away from 0. This relationship between \(-4\) and \(+4\) is an important one. Such numbers are known as **opposites** or **additive inverses** of each other.

**Opposites or Additive Inverses**

Two numbers that are the same distance from 0 but lie on opposite sides of 0 are called **opposites** or **additive inverses** of each other.

**Example 9** Find the opposite or additive inverse of each number.

a. 5  
   b. \(-6\)  
   c. \(\frac{1}{2}\)  
   d. \(-4.5\)

**Solution**

a. The opposite of 5 is \(-5\). Notice that 5 and \(-5\) are on opposite sides of 0 when plotted on a number line and are equal distances away.

   b. The opposite of \(-6\) is 6.

   c. The opposite of \(\frac{1}{2}\) is \(-\frac{1}{2}\).

   d. The opposite of \(-4.5\) is 4.5.

**Practice 9** Find the opposite or additive inverse of each number.

a. \(-\frac{5}{9}\)  
   b. 8  
   c. 6.2  
   d. \(-3\)
We use the symbol “−” to represent the phrase “the opposite of” or “the additive inverse of.” In general, if \( a \) is a number, we write the opposite or additive inverse of \( a \) as \(-a\). We know that the opposite of \(-3\) is 3. Notice that this translates as

\[
\text{the opposite of } -3 \quad \text{is} \quad 3
\]

This is true in general. If \( a \) is a number, then \(-1 - a = a\).

**Example 10**  Simplify each expression.

\[
\begin{align*}
a. & \quad -(\text{-}10) \quad \text{b.} \quad -(\frac{1}{2}) \quad \text{c.} \quad -(\text{-}2x) \quad \text{d.} \quad -|6|
\end{align*}
\]

**Solution**

\[
\begin{align*}
a. & \quad -(\text{-}10) = 10 \\
b. & \quad -(\frac{1}{2}) = \frac{1}{2} \\
c. & \quad -(\text{-}2x) = 2x \\
d. & \quad \text{Since } -|6| = 6, \text{ then } -|6| = -6.
\end{align*}
\]

**Practice 10**  Simplify each expression.

\[
\begin{align*}
a. & \quad -|\text{-}15| \\
b. & \quad -(\frac{3}{5}) \\
c. & \quad -(\text{-}5y) \\
d. & \quad -(\text{-}8)
\end{align*}
\]

Let’s discover another characteristic about opposites. Notice that the sum of a number and its opposite is 0.

\[
\begin{align*}
10 + (\text{-}10) & = 0 \\
-3 + 3 & = 0 \\
\frac{1}{2} + (-\frac{1}{2}) & = 0
\end{align*}
\]

In general, we can write the following:

\[
\text{The sum of a number } a \text{ and its opposite } -a \text{ is 0.} \\
\text{ } a + (-a) = 0
\]

This is why opposites are also called additive inverses. Notice that this also means that the opposite of 0 is then 0 since \(0 + 0 = 0\).

**Vocabulary, Readiness & Video Check**

*Use the choices below to fill in each blank. Not all choices will be used.*

- a positive number
- \( n \)
- opposites
- a negative number
- \( 0 \)
- \(-n\)

1. Two numbers that are the same distance from 0 but lie on opposite sides of 0 are called ________________.
2. If \( n \) is a number, then \( n + (-n) = \) ________________.
3. If \( n \) is a number, then \(-(-n) = \) ________________.
4. The sum of two negative numbers is always ________________.
Watch the section lecture video and answer the following questions.

5. Complete this statement based on the lecture given before Example 1. To add two numbers with the same sign, add their ________ and use their common sign as the sign of the sum.

6. What is the sign of the sum in Example 6 and why?

7. What is the real life application of negative numbers used in Example 9? The answer to Example 9 is −6. What does this number mean in the context of the problem?

8. Example 12 illustrates the idea that if a is a real number, the opposite of −a is a. Example 13 looks similar to Example 12, but it’s actually quite different. Explain the difference.

### 1.5 Exercise Set

**MIXED PRACTICE**

Add. See Examples 1 through 7.

1. $-6 + 3$
2. $9 + (-12)$
3. $-6 + (-8)$
4. $-6 + (-14)$
5. $8 + (-7)$
6. $6 + (-4)$
7. $-14 + 2$
8. $-10 + 5$
9. $-2 + (-3)$
10. $-7 + (-4)$
11. $-9 + (-3)$
12. $7 + (-5)$
13. $-7 + 3$
14. $-5 + 9$
15. $10 + (-3)$
16. $8 + (-6)$
17. $5 + (-7)$
18. $3 + (-6)$
19. $-16 + 16$
20. $23 + (-23)$
21. $27 + (-46)$
22. $53 + (-37)$
23. $-18 + 49$
24. $-26 + 14$
25. $-33 + (-14)$
26. $-18 + (-26)$
27. $6.3 + (-8.4)$
28. $9.2 + (-11.4)$
29. $|8| + (-16)$
30. $|-6| + (-61)$
31. $117 + (-79)$
32. $144 + (-88)$
33. $-9.6 + (-3.5)$
34. $-6.7 + (-7.6)$
35. $-\frac{3}{8} + \frac{5}{8}$
36. $-\frac{5}{12} + \frac{7}{12}$
37. $-\frac{7}{16} + \frac{1}{4}$
38. $-\frac{5}{9} + \frac{1}{3}$
39. $-\frac{7}{10} + \left(\frac{-3}{5}\right)$
40. $-\frac{5}{6} + \left(\frac{-2}{3}\right)$
41. $-15 + 9 + (-2)$
42. $-9 + 15 + (-5)$
43. $-21 + (-16) + (-22)$
44. $-18 + (-6) + (-40)$
45. $-23 + 16 + (-2)$
46. $-14 + (-3) + 11$
47. $5 + (-10)$
48. $7 + (-17)$
49. $6 + (-4) + 9$
50. $8 + (-2) + 7$
51. $[-17 + (-4)] + [-12 + 15]$
52. $[-2 + (-7)] + [-11 + 22]$
53. $[9 + (-12)] + [-16]$
54. $|43 + (-73)| + |[-20]|
55. $-1.3 + [0.5 + (-0.3) + 0.4]$
56. $-3.7 + [0.1 + (-0.6) + 8.1]$

Solve. See Example 8.

57. The low temperature in Anoka, Minnesota, was $-15^\circ$ last night. During the day, it rose only $9^\circ$. Find the high temperature for the day.

58. On January 2, 1943, the temperature was $-4^\circ$ at 7:30 a.m. in Spearfish, South Dakota. Incredibly, it got $49^\circ$ warmer in the next 2 minutes. To what temperature did it rise by 7:32?

59. The lowest point in Africa is $-512$ feet at Lake Assal in Djibouti. If you are standing at a point 658 feet above Lake Assal, what is your elevation? (Source: Microsoft Encarta)

60. The lowest elevation in Australia is $-52$ feet at Lake Eyre. If you are standing at a point 439 feet above Lake Eyre, what is your elevation? (Source: National Geographic Society)
A negative net income results when a company spends more money than it brings in.

61. Johnson Outdoors Inc. had the following quarterly net incomes during its 2014 fiscal year. (Source: Johnsonoutdoors.com)

<table>
<thead>
<tr>
<th>Quarter of Fiscal 2014</th>
<th>Net Income (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>$7.4</td>
</tr>
<tr>
<td>Second</td>
<td>$4.7</td>
</tr>
<tr>
<td>Third</td>
<td>$-0.8</td>
</tr>
<tr>
<td>Fourth</td>
<td>$-2.2</td>
</tr>
</tbody>
</table>

What was the total net income for fiscal year 2014?

62. LeapFrog Enterprises Inc. had the following quarterly net incomes during its 2013 fiscal year. (Source: Leapfroginvestors.com)

<table>
<thead>
<tr>
<th>Quarter of Fiscal 2013</th>
<th>Net Income (in millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>$3.0</td>
</tr>
<tr>
<td>Second</td>
<td>$-3.2</td>
</tr>
<tr>
<td>Third</td>
<td>$26.4</td>
</tr>
<tr>
<td>Fourth</td>
<td>$63.9</td>
</tr>
</tbody>
</table>

What was the total net income for fiscal year 2013?

In golf, scores that are under par for the entire round are shown as negative scores; positive scores are shown for scores that are over par, and 0 is par.

63. Austin Ernst was the winner of the 2014 Portland Classic in Oregon. Her scores were −3, −3, −3, and −5. What was her overall score? (Source: Ladies Professional Golf Association)

64. During the 2015 Hyundai Tournament of champions in Maui, Hawaii, Patrick Reed won with scores of −6, −4, −5, and −6. What was his overall score? (Source: Professional Golfers’ Association of America)

Find each additive inverse or opposite. See Example 9.

65. 6  66. 4  67. −2

68. −8  69. 0  70. $\frac{1}{4}$

71. −6  72. −11

Simplify each of the following. See Example 10.

73. −|−2| 74. −(−3) 75. −|0|

76. $\frac{2}{3}$ 77. −$\frac{2}{3}$ 78. −(−7)

Decide whether the given number is a solution of the given equation.

79. Is −4 a solution of $x + 9 = 5$?

80. Is 10 a solution of $7 = −x + 3$?

81. Is −1 a solution of $y + (−3) = −7$?

82. Is −6 a solution of $1 = y + 7$?

CONCEPT EXTENSIONS

The following bar graph shows each month’s average daily low temperature in degrees Fahrenheit for Barrow, Alaska. Use this graph to answer Exercises 83 through 88.

83. For what month is the graphed temperature the highest?

84. For what month is the graphed temperature the lowest?

85. For what month is the graphed temperature positive and closest to 0°?

86. For what month is the graphed temperature negative and closest to 0°?

87. Find the average of the temperatures shown for the months of April, May, and October. (To find the average of three temperatures, find their sum and divide by 3)

88. Find the average of the temperatures shown for the months of January, September, and October.

Each calculation below is incorrect. Find the error and correct it. See the Concept Check in this section.

89. $7 + (−10) = 17$  90. $−4 + 14 = 18$

91. $−10 + (−12) = −120$  92. $−15 + (−12) = 32$

If $a$ is a positive number and $b$ is a negative number, fill in the blanks with the words positive or negative.

93. $−a$ is __________  94. $−b$ is __________

95. $a + a$ is __________  96. $b + b$ is __________

For Exercises 97 through 100, determine whether each statement is true or false.

97. The sum of two negative numbers is always a negative number.

98. The sum of two positive numbers is always a positive number.

99. The sum of a positive number and a negative number is always a negative number.

100. The sum of zero and a negative number is always a negative number.

101. In your own words, explain how to find the opposite of a number.

102. In your own words, explain why 0 is the only number that is its own opposite.

103. Explain why adding a negative number to another negative number always gives a negative sum.

104. When a positive and a negative number are added, sometimes the sum is positive, sometimes it is zero, and sometimes it is negative. Explain why and when this happens.
Chapter 1
Review of Real Numbers

1.6 Subtracting Real Numbers

**OBJECTIVES**

1. Subtract Real Numbers.
2. Add and Subtract Real Numbers.
3. Evaluate Algebraic Expressions Using Real Numbers.
4. Solve Applications That Involve Subtraction of Real Numbers.
5. Find Complementary and Supplementary Angles.

**OBJECTIVE 1 Subtracting Real Numbers**

Now that addition of signed numbers has been discussed, we can explore subtraction. We know that $9 - 7 = 2$. Notice that $9 + (-7) = 2$ also. This means that

$$9 - 7 = 9 + (-7)$$

Notice that the difference of 9 and 7 is the same as the sum of 9 and the opposite of 7. In general, we have the following.

**Subtracting Two Real Numbers**

If $a$ and $b$ are real numbers, then $a - b = a + (-b)$.

In other words, to find the difference of two numbers, add the first number to the opposite of the second number.

**EXAMPLE 1** Subtract.

a. $-13 - 4$

b. $5 - (-6)$

c. $3 - 6$

d. $-1 - (-7)$

**Solution**

a. $-13 - 4 = -13 + (-4)$

Add $-13$ to the opposite of $4$, which is $-4$.

$$= -17$$

b. $5 - (-6) = 5 + 6$

Add $5$ to the opposite of $-6$, which is $6$.

$$= 11$$

c. $3 - 6 = 3 + (-6)$

Add $3$ to the opposite of $6$, which is $-6$.

$$= -3$$

d. $-1 - (-7) = -1 + (7) = 6$

**PRACTICE** Subtract.

a. $-7 - 6$

b. $-8 - (-1)$

c. $9 - (-3)$

d. $5 - 7$

**Helpful Hint**

Study the patterns indicated.

No change

Change to addition.

Change to opposite.

$$5 - 11 = 5 + (-11) = -6$$

$$-3 - 4 = -3 + (-4) = -7$$

$$7 - (-1) = 7 + (1) = 8$$

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EXAMPLE 2 Subtract.

a. \( 5.3 - (-4.6) \)

b. \( \frac{3}{10} - \frac{5}{10} \)

c. \( -\frac{2}{3} - \left( -\frac{4}{5} \right) \)

Solution

a. \( 5.3 - (-4.6) = 5.3 + (4.6) = 9.9 \)

b. \( -\frac{3}{10} - \frac{5}{10} = -\frac{3}{10} + (-\frac{5}{10}) = -\frac{8}{10} = -\frac{4}{5} \)

c. \( -\frac{2}{3} - \left( -\frac{4}{5} \right) = -\frac{2}{3} + \frac{4}{5} = -\frac{10}{15} + \frac{12}{15} = \frac{2}{15} \)

The common denominator is 15.

PRACTICE Subtract.

a. \( 8.4 - (-2.5) \)

b. \( -\frac{5}{8} - \left( -\frac{1}{8} \right) \)

c. \( -\frac{3}{4} - \frac{1}{5} \)

EXAMPLE 3 Subtract 8 from \(-4\).

Solution Be careful when interpreting this: The order of numbers in subtraction is important. 8 is to be subtracted from \(-4\).

\[ -4 - 8 = -4 + (-8) = -12 \]

PRACTICE Subtract 5 from \(-2\).

OBJECTIVE Adding and Subtracting Real Numbers

If an expression contains additions and subtractions, just write the subtractions as equivalent additions. Then simplify from left to right.

EXAMPLE 4 Simplify each expression.

a. \( -14 - 8 + 10 - (-6) \)

b. \( 1.6 - (-10.3) + (-5.6) \)

Solution

a. \( -14 - 8 + 10 - (-6) = -14 + (-8) + 10 + 6 \)

\[ = -6 \]

b. \( 1.6 - (-10.3) + (-5.6) = 1.6 + 10.3 + (-5.6) \)

\[ = 6.3 \]

PRACTICE Simplify each expression.

a. \( -15 - 2 - (-4) + 7 \)

b. \( 3.5 + (-4.1) - (-6.7) \)

When an expression contains parentheses and brackets, remember the order of operations. Start with the innermost set of parentheses or brackets and work your way outward.
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Chapter 1 Review of Real Numbers

Example 5 Simplify each expression.

a. \(-3 + [(-2 - 5) - 2]\)  
b. \(2^3 - |10| + [-6 - (-5)]\)

Solution

a. Start with the innermost set of parentheses. Rewrite \(-2 - 5\) as a sum.

\[-3 + [(-2 - 5) - 2] = -3 + [(-2 + (-5)) - 2] = -3 + [(-7) - 2] = -3 + [-7 + (-2)] = -3 + [-9] = -12\]

b. Start simplifying the expression inside the brackets by writing \(-6 - (-5)\) as a sum.

\[2^3 - |10| + [-6 - (-5)] = 2^3 - |10| + [-6 + 5] = 2^3 - |10| + [-1] = 8 - 10 + (-1) = 8 + (-10) + (-1) = -2 + (-1) = -3\]

Practice 5 Simplify each expression.

a. \(-4 + [(-8 - 3) - 5]\)  
b. \(|-13| - 3^2 + [2 - (-7)]\)

Objective 3 Evaluating Algebraic Expressions

Knowing how to evaluate expressions for given replacement values is helpful when checking solutions of equations and when solving problems whose unknowns satisfy given expressions. The next example illustrates this.

Example 6 Find the value of each expression when \(x = 2\) and \(y = -5\).

a. \(\frac{x - y}{12 + x}\)  
b. \(x^2 - 3y\)

Solution

a. Replace \(x\) with 2 and \(y\) with \(-5\). Be sure to put parentheses around \(-5\) to separate signs. Then simplify the resulting expression.

\[\frac{x - y}{12 + x} = \frac{2 - (-5)}{12 + 2} = \frac{2 + 5}{14} = \frac{7}{14} = \frac{1}{2}\]

b. Replace \(x\) with 2 and \(y\) with \(-5\) and simplify.

\[x^2 - 3y = 2^2 - 3(-5) = 4 - 3(-5) = 4 + 15 = 19\]
Section 1.6 Subtracting Real Numbers

Practice 6 Find the value of each expression when \( x = -3 \) and \( y = 4 \).

\[ a. \quad \frac{7 - x}{2y + x} \]

\[ b. \quad y^2 + x \]

Helpful Hint

For additional help when replacing variables with replacement values, first place parentheses around any variables.

For Example 6b on the previous page, we have

\[
\begin{align*}
\frac{x^2 - 3y}{(x^2 - 3y)} &= \frac{(2)^2 - 3(-5)}{4 - 3(-5)} = 4 - 3(-5) = 4 - (-15) = 4 + 15 = 19
\end{align*}
\]

Place parentheses Replace variables around variables with values

Objective 4 Solving Applications by Subtracting Real Numbers

One use of positive and negative numbers is in recording altitudes above and below sea level, as shown in the next example.

Example 7 Finding a Change in Elevation

The highest point in the United States is the top of Mount McKinley, at a height of 20,320 feet above sea level. The lowest point is Death Valley, California, which is 282 feet below sea level. How much higher is Mount McKinley than Death Valley? (Source: U.S. Geological Survey)

Solution: To find “how much higher,” we subtract. Don’t forget that since Death Valley is 282 feet below sea level, we represent its height by \(-282\). Draw a diagram to help visualize the problem.

![Diagram of Mount McKinley and Death Valley]

In words: how much higher is Mt. McKinley

\downarrow \quad \downarrow \quad \downarrow

Translate: how much higher is Mt. McKinley

\downarrow \quad \downarrow \quad \downarrow

\begin{align*}
\text{height of Mt. McKinley} &= 20,320 \\
\text{minus} \quad \text{height of Death Valley} &= -(-282) \\
\text{Thus, Mount McKinley is} \quad \text{20,602 feet} \quad \text{higher than Death Valley.}
\end{align*}

Practice 7 On Tuesday morning, a bank account balance was $282. On Thursday, the account balance had dropped to $-75. Find the overall change in this account balance.
A knowledge of geometric concepts is needed by many professionals, such as doctors, carpenters, electronic technicians, gardeners, machinists, and pilots, just to name a few. With this in mind, we review the geometric concepts of complementary and supplementary angles.

**Complementary and Supplementary Angles**

Two angles are **complementary** if their sum is $90^\circ$.

Two angles are **supplementary** if their sum is $180^\circ$.

**EXAMPLE 8** Find each unknown complementary or supplementary angle.

**a.**

These angles are complementary, so their sum is $90^\circ$. This means that $x$ is $90^\circ - 38^\circ$.

\[
x = 90^\circ - 38^\circ = 52^\circ
\]

**b.** These angles are supplementary, so their sum is $180^\circ$. This means that $y$ is $180^\circ - 62^\circ$.

\[
y = 180^\circ - 62^\circ = 118^\circ
\]

**Practice 8** Find each unknown complementary or supplementary angle.

**a.**

**b.**

**Vocabulary, Readiness & Video Check**

Translate each phrase. Let $x$ represent “a number.” Use the choices below to fill in each blank.

$7 - x$  
$x - 7$

1. 7 minus a number  
2. 7 subtracted from a number  
3. A number decreased by 7  
4. 7 less a number  
5. A number less than 7  
6. A number subtracted from 7

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**Multiple choice: Select the correct lettered response following each exercise.**

7. To evaluate \( x - y \) for \( x = -10 \) and \( y = -14 \), we replace \( x \) with \( -10 \) and \( y \) with \( -14 \) and evaluate \[ \boxed{\text{a. } 10 - 14 \quad \text{b. } -10 - 14 \quad \text{c. } -14 - 10 \quad \text{d. } -10 - (-14)} \].

8. The expression \(-5 - 10\) equals \[ \boxed{\text{a. } 5 - 10 \quad \text{b. } 5 + 10 \quad \text{c. } -5 + (-10) \quad \text{d. } 10 - 5} \].

**Martin-Gay Interactive Videos**

Watch the section lecture video and answer the following questions.

9. Complete this statement based on the lecture given before Example 1. To subtract two real numbers, change the operation to _______ and take the _______ of the second number.

10. When simplifying Example 5, what is the result of the first step and why is the expression rewritten in this way?

11. In Example 7, why are you told to be especially careful when working with the replacement value in the numerator?

12. For Example 8, why is the overall vertical change represented as a negative number?

13. The definition of supplementary angles is given just before Example 9. Explain how this definition is used to solve Example 9.

---

**1.6 Exercise Set**

**MIXED PRACTICE**

Subtract. See Examples 1 and 2.

\[ \begin{align*}
1. \quad & -6 - 4 \\
2. \quad & -12 - 8 \\
3. \quad & 4 - 9 \\
4. \quad & 8 - 11 \\
5. \quad & 16 - (-3) \\
6. \quad & 12 - (-5) \\
7. \quad & \frac{1}{2} - \frac{1}{3} \\
8. \quad & \frac{3}{4} - \frac{7}{8} \\
9. \quad & -16 - (-18) \\
10. \quad & -6 - 5 \\
11. \quad & 7 - (-4) \\
12. \quad & -8 - 4 \\
13. \quad & -6 - (-11) \\
14. \quad & 3 - (-6) \\
15. \quad & -4 - (-16) \\
16. \quad & 15 - (-33) \\
17. \quad & 16 - (-21) \\
18. \quad & 8.3 - 11.2 \\
19. \quad & -44 - 27 \\
20. \quad & -36 - 51 \\
21. \quad & -21 - (-21) \\
22. \quad & -17 - (-17) \\
23. \quad & -2.6 - (-6.7) \\
24. \quad & -6.1 - (-5.3) \\
25. \quad & \frac{-3}{11} - \left( \frac{-5}{11} \right) \\
26. \quad & -\frac{4}{7} - \left( -\frac{1}{7} \right) \\
27. \quad & \frac{-1}{6} + \frac{3}{4} \\
28. \quad & \frac{1}{2} - \frac{1}{8} \\
29. \quad & 8.3 - (-0.62) \\
30. \quad & 4.3 - (-0.87)
\end{align*} \]

---

**TRANSLATING**

Translate each phrase to an expression and simplify. See Example 3.

\[ \begin{align*}
33. \quad & \text{Subtract } -5 \text{ from } 8. \\
34. \quad & \text{Subtract } 3 \text{ from } -2. \\
35. \quad & \text{Subtract } -1 \text{ from } -6. \\
36. \quad & \text{Subtract } 17 \text{ from } 1. \\
37. \quad & \text{Subtract } 8 \text{ from } 7. \\
38. \quad & \text{Subtract } 9 \text{ from } -4. \\
39. \quad & \text{Decrease } -8 \text{ by } 15. \\
40. \quad & \text{Decrease } 11 \text{ by } -14. \\
41. \quad & -10 - (-8) + (-4) - 20 \\
42. \quad & -16 - (-3) + (-11) - 14 \\
43. \quad & 5 - 9 + (-4) - 8 - 8 \\
44. \quad & 7 - 12 + (-5) - 2 + (-2) \\
45. \quad & -6 - (2 - 11) \\
46. \quad & -9 - (3 - 8) \\
47. \quad & 3^3 - 8 \cdot 9 \\
48. \quad & 2^3 - 6 \cdot 3 \\
49. \quad & 2 - 3(8 - 6) \\
50. \quad & 4 - 6(7 - 3) \\
51. \quad & (3 - 6) + 2^2 \\
52. \quad & (2 - 3) + 5^2 \\
53. \quad & -2 + [(8 - 11) - (-2 - 9)] \\
54. \quad & -5 + [(4 - 15) - (-6) - 8] \\
55. \quad & [-3] + 2^2 + [-4 - (-6)] \\
56. \quad & [-2] + 6^2 + (-3 - 8)
\end{align*} \]
Evaluate each expression when \( x = -5, y = 4, \) and \( t = 10. \) See Example 6.

57. \( x - y \) 
58. \( y - x \) 
59. \( |x| + 2t - 8y \) 
60. \( |x + t - 7y| \) 
61. \( \frac{9 - x}{y + 6} \) 
62. \( \frac{15 - x}{y + 2} \) 
63. \( y^2 - x \) 
64. \( t^2 - x \) 
65. \( \frac{|x - (-10)|}{2t} \) 
66. \( \frac{|5y - x|}{6t} \)

Solve. See Example 7.

67. Within 24 hours in 1916, the temperature in Browning, Montana, fell from \( 44^\circ F \) to \(-56^\circ F. \) How large a drop in temperature was this?
68. Much of New Orleans is below sea level. If George descends 12 feet from an elevation of 5 feet above sea level, what is his new elevation?
69. The coldest temperature ever recorded on Earth was \(-129^\circ F. \) In Antarctica, the warmest temperature ever recorded was \(134^\circ F. \) How many degrees warmer is \(134^\circ F\) than \(-129^\circ F?\) (Source: World Meteorological Organization)
70. The coldest temperature ever recorded in the United States was \(-80^\circ F. \) In Alaska. The warmest temperature ever recorded was \(134^\circ F. \) How many degrees warmer is \(134^\circ F\) than \(-80^\circ F?\) (Source: The World Almanac)
71. Mauna Kea in Hawaii has an elevation of 13,796 feet above sea level. The Mid-America Trench in the Pacific Ocean has an elevation of 21,857 feet below sea level. Find the difference in elevation between those two points. (Source: National Geographic Society and Defense Mapping Agency)
72. A woman received a statement of her charge account at Old Navy. She spent \$93\) on purchases last month. She returned an \$18\) top because she didn't like the color. She also returned a \$26\) nightshirt because it was damaged. What does she actually owe on her account?
73. A commercial jetliner hits an air pocket and drops 250 feet. After climbing 120 feet, it drops another 178 feet. What is its overall vertical change?
74. In some card games, it is possible to have a negative score. Lavonne Schultz currently has a score of 15 points. She then loses 24 points. What is her new score?
75. The highest point in Africa is Mt. Kilimanjaro, Tanzania, at an elevation of 19,340 feet. The lowest point is Lake Assal, Djibouti, at 512 feet below sea level. How much higher is Mt. Kilimanjaro than Lake Assal? (Source: National Geographic Society)

76. The airport in Bishop, California, is at an elevation of 4101 feet above sea level. The nearby Furnace Creek Airport in Death Valley, California, is at an elevation of 226 feet below sea level. How much higher in elevation is the Bishop Airport than the Furnace Creek Airport? (Source: National Climatic Data Center)

Find each unknown complementary or supplementary angle. See Example 8.

77. \( \triangle \) 78. \( \triangle \)

79. \( \triangle \) 80. \( \triangle \)

Decide whether the given number is a solution of the given equation.

81. Is \(-4\) a solution of \( x - 9 = 5\)?
82. Is \(3\) a solution of \( x - 10 = -7\)?
83. Is \(-2\) a solution of \(-x + 6 = -x - 1\)?
84. Is \(-10\) a solution of \(-x - 6 = -x - 1\)?
85. Is \(2\) a solution of \(-x - 13 = -15\)?
86. Is \(5\) a solution of \(4 = 1 - x\)?

MIXED PRACTICE—TRANSLATING (SECTIONS 1.5, 1.6)

Translate each phrase to an algebraic expression. Use “\( x \)” to represent “a number.”

87. The sum of \(-5\) and a number.
88. The difference of \(-3\) and a number.
89. Subtract a number from \(-20\).
90. Add a number and \(-36\).

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### Concept Extensions

Recall the bar graph from Section 1.5. It shows each month's average daily low temperature in degrees Fahrenheit for Barrow, Alaska. Use this graph to answer Exercises 91 through 94.

![Bar graph for Barrow, Alaska](image)

**Data from National Climatic Data Center**

91. Record the monthly increases and decreases in the low temperature from the previous month.

<table>
<thead>
<tr>
<th>Month</th>
<th>Monthly Increase or Decrease (from the previous month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>February</td>
<td></td>
</tr>
<tr>
<td>March</td>
<td></td>
</tr>
<tr>
<td>April</td>
<td></td>
</tr>
<tr>
<td>May</td>
<td></td>
</tr>
<tr>
<td>June</td>
<td></td>
</tr>
</tbody>
</table>

92. Record the monthly increases and decreases in the low temperature from the previous month.

<table>
<thead>
<tr>
<th>Month</th>
<th>Monthly Increase or Decrease (from the previous month)</th>
</tr>
</thead>
<tbody>
<tr>
<td>July</td>
<td></td>
</tr>
<tr>
<td>August</td>
<td></td>
</tr>
<tr>
<td>September</td>
<td></td>
</tr>
<tr>
<td>October</td>
<td></td>
</tr>
<tr>
<td>November</td>
<td></td>
</tr>
<tr>
<td>December</td>
<td></td>
</tr>
</tbody>
</table>

93. Which month had the greatest increase in temperature?

94. Which month had the greatest decrease in temperature?

95. Find two numbers whose difference is \(-5\).

96. Find two numbers whose difference is \(-9\).

Each calculation below is incorrect. Find the error and correct it.

97. \[ 9 - (-7) = 2 \]
98. \[ -4 - 8 = 4 \]
99. \[ 10 - 30 = 20 \]
100. \[ -3 - (-10) = -13 \]

If \(p\) is a positive number and \(n\) is a negative number, determine whether each statement is true or false. Explain your answer.

101. \(p - n\) is always a positive number.
102. \(n - p\) is always a negative number.
103. \(|n| - |p|\) is always a positive number.
104. \(|n| - p\) is always a positive number.

Without calculating, determine whether each answer is positive or negative. Then use a calculator to find the exact difference.

105. \(56,875 - 87,262\)
106. \(4,362 - 7,0086\)

### Integrated Review

#### Operations on Real Numbers

**Sections 1.1–1.6**

Answer the following with positive, negative, or 0.

1. The opposite of a positive number is a ______ number.
2. The sum of two negative numbers is a ______ number.
3. The absolute value of a negative number is a ______ number.
4. The absolute value of zero is ______.
5. The reciprocal of a positive number is a ______ number.
6. The sum of a number and its opposite is ______.
7. The absolute value of a positive number is a ______ number.
8. The opposite of a negative number is a ______ number.

Fill in the chart:

<table>
<thead>
<tr>
<th>Number</th>
<th>Opposite</th>
<th>Absolute Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>9. (\frac{1}{7})</td>
<td>(-\frac{1}{7})</td>
<td>(\frac{1}{7})</td>
</tr>
<tr>
<td>10. (-\frac{12}{5})</td>
<td>(\frac{12}{5})</td>
<td>(\frac{12}{5})</td>
</tr>
<tr>
<td>11.</td>
<td>(-3)</td>
<td>(-3)</td>
</tr>
<tr>
<td>12.</td>
<td>(\frac{9}{11})</td>
<td>(\frac{9}{11})</td>
</tr>
</tbody>
</table>

Perform each indicated operation and simplify.

13. \(-19 + (\neg23)\)
14. \(7 - (-3)\)
15. \(-15 + 17\)
16. \(-8 - 10\)
1.7 Multiplying and Dividing Real Numbers

**OBJECTIVES**

1. Multiply Real Numbers.
2. Find the Reciprocal of a Real Number.
3. Divide Real Numbers.
4. Evaluate Expressions Using Real Numbers.
5. Solve Applications That Involve Multiplication or Division of Real Numbers.

**OBJECTIVE 1 Multiplying Real Numbers**

In this section, we discover patterns for multiplying and dividing real numbers. To discover sign rules for multiplication, recall that multiplication is repeated addition. Thus 3 \cdot 2 means that 2 is an addend 3 times. That is, 

\[
2 + 2 + 2 = 3 \cdot 2
\]

which equals 6. Similarly, 3 \cdot (-2) means -2 is an addend 3 times. That is, 

\[
(-2) + (-2) + (-2) = 3 \cdot (-2)
\]

Since (-2) + (-2) + (-2) = -6, 3 \cdot (-2) = -6. This suggests that the product of a positive number and a negative number is a negative number.

What about the product of two negative numbers? To find out, consider the following pattern.

\[
\begin{align*}
-3 \cdot 2 &= -6 \\
-3 \cdot 1 &= -3 \\
-3 \cdot 0 &= 0
\end{align*}
\]

This pattern continues as 

\[
\begin{align*}
-3 \cdot -1 &= 3 \\
-3 \cdot -2 &= 6
\end{align*}
\]

This suggests that the product of two negative numbers is a positive number.

**Multiplying Real Numbers**

1. The product of two numbers with the same sign is a positive number.
2. The product of two numbers with different signs is a negative number.
We know that any whole number multiplied by zero equals zero. This remains true for real numbers.

**Example 1**
Multiply.

a. \((-8)(4)\)

**Solution**

\[-8(4) = -32\]

b. \(14(-1)\)

c. \(-9(-10)\)

**Practice 1**
Multiply.

a. \(8(-5)\)

b. \((-3)(-4)\)

c. \((-6)(9)\)

We know that any whole number multiplied by zero equals zero. This remains true for real numbers.

**Zero as a Factor**

If \(b\) is a real number, then \(b \cdot 0 = 0\). Also, \(0 \cdot b = 0\).

**Example 2**
Perform the indicated operations.

a. \((7)(0)(-6)\)

b. \((-2)(-3)(-4)\)

c. \((-1)(5)(-9)\)

d. \((-4)(-11) - (5)(-2)\)

**Solution**

a. By the order of operations, we multiply from left to right. Notice that, because one of the factors is 0, the product is 0.

\[(7)(0)(-6) = 0(-6) = 0\]

b. Multiply two factors at a time, from left to right.

\[(-2)(-3)(-4) = (6)(-4)\]

Multiply \(-2)(-3)\).

\[= -24\]

c. Multiply from left to right.

\[(-1)(5)(-9) = (-5)(-9)\]

Multiply \(-1)(5)\).

\[= 45\]

d. Follow the rules for order of operations.

\[(-4)(-11) - (5)(-2) = 44 - (-10)\]

Find each product.

\[= 44 + 10\]

Add 44 to the opposite of \(-10\).

\[= 54\]

Add.

**Practice 2**
Perform the indicated operations.

a. \((-1)(-5)(-6)\)

b. \((-3)(-2)(4)\)

c. \((-4)(0)(5)\)

d. \((-2)(-3) - (4)(5)\)

**Helpful Hint**

You may have noticed from the example that if we multiply:

- an even number of negative numbers, the product is positive.
- an odd number of negative numbers, the product is negative.

**Concept Check**

What is the sign of the product of five negative numbers? Explain.

Answer to Concept Check:

negative
Chapter 1  Review of Real Numbers

Now that we know how to multiply positive and negative numbers, let’s see how we find the values of \(-4^2\) and \(-4^2\), for example. Although these two expressions look similar, the difference between the two is the parentheses. In \(-4^2\), the parentheses tell us that the base, or repeated factor, is \(-4\).

\[-4^2 = (-4)(-4) = 16\] The base is \(-4\).
\[-4^2 = -(4\cdot 4) = -16\] The base is 4.

### Example 4

Evaluate.

a. \((-2)^3\) \hspace{1cm} b. \(-2^3\) \hspace{1cm} c. \((-3)^2\) \hspace{1cm} d. \(-3^2\)

**Solution**

a. \((-2)^3 = (-2)(-2)(-2) = -8\) The base is \(-2\).

b. \(-2^3 = -(2\cdot 2\cdot 2) = -8\) The base is 2.

c. \((-3)^2 = (-3)(-3) = 9\) The base is \(-3\).

d. \(-3^2 = -(3\cdot 3) = -9\) The base is 3.

### Example 3

Multiply.

a. \((-1.2)(0.05)\) \hspace{1cm} b. \(\frac{2}{3}\cdot \left(-\frac{7}{10}\right)\) \hspace{1cm} c. \(\left(-\frac{4}{5}\right)(-20)\)

**Solution**

a. The product of two numbers with different signs is negative.

\[(-1.2)(0.05) = -[(1.2)(0.05)]\]
\[= -0.06\]

b. \(\frac{2}{3}\cdot \left(-\frac{7}{10}\right) = \frac{2\cdot 7}{3\cdot 10} = \frac{14}{30} = \frac{7}{15}\)

c. \(\left(-\frac{4}{5}\right)(-20) = \frac{4\cdot 20}{5\cdot 1} = \frac{80}{5} = 16\) or 16

### Practice

3 Multiply.

a. \((0.23)(-0.2)\) \hspace{1cm} b. \(-\frac{3}{5}\cdot \left(\frac{4}{9}\right)\) \hspace{1cm} c. \(\left(-\frac{7}{12}\right)(-24)\)

Now that we know how to multiply positive and negative numbers, let’s see how we find the values of \((-4)^2\) and \(-4^2\), for example. Although these two expressions look similar, the difference between the two is the parentheses. In \((-4)^2\), the parentheses tell us that the base, or repeated factor, is \(-4\). In \(-4^2\), only 4 is the base. Thus,

\[-4^2 = (-4)(-4) = 16\] The base is \(-4\).
\[-4^2 = -(4\cdot 4) = -16\] The base is 4.

4 Evaluate.

a. \((-6)^2\) \hspace{1cm} b. \(-6^2\) \hspace{1cm} c. \((-4)^3\) \hspace{1cm} d. \(-4^3\)
In other words, the quotient of two real numbers is the product of the first number and the multiplicative inverse or reciprocal of the second number.

**Reciprocals or Multiplicative Inverses**

Two numbers whose product is 1 are called reciprocals or multiplicative inverses of each other.

Notice that 0 has no multiplicative inverse since 0 multiplied by any number is never 1 but always 0.

**Example 5** Find the reciprocal of each number.

a. \(22\)  
b. \(\frac{3}{16}\)  
c. \(-10\)  
d. \(-\frac{9}{13}\)

**Solution**

a. The reciprocal of 22 is \(\frac{1}{22}\) since \(22 \cdot \frac{1}{22} = 1\).

b. The reciprocal of \(\frac{3}{16}\) is \(\frac{16}{3}\) since \(\frac{3}{16} \cdot \frac{16}{3} = 1\).

c. The reciprocal of \(-10\) is \(-\frac{1}{10}\).

**Practice**  
Find the reciprocal of each number.

a. \(\frac{8}{3}\)  
b. \(15\)  
c. \(-\frac{2}{7}\)  
d. \(-5\)

**Objective 3** Dividing Real Numbers

We may now write a quotient as an equivalent product.

**Quotient of Two Real Numbers**

If \(a\) and \(b\) are real numbers and \(b\) is not 0, then

\[a \div b = \frac{a}{b} = a \cdot \frac{1}{b}\]

In other words, the quotient of two real numbers is the product of the first number and the multiplicative inverse or reciprocal of the second number.

**Example 6** Use the definition of the quotient of two numbers to divide.

a. \(-18 \div 3\)  
b. \(-14 \div -2\)  
c. \(\frac{20}{-4}\)

**Solution**

a. \(-18 \div 3 = -18 \cdot \frac{1}{3} = -6\)  
b. \(-14 \div -2 = -14 \cdot -\frac{1}{2} = 7\)  
c. \(\frac{20}{-4} = 20 \cdot -\frac{1}{4} = -5\)

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Since the quotient \( a \div b \) can be written as the product \( a \cdot \frac{1}{b} \), it follows that sign patterns for dividing two real numbers are the same as sign patterns for multiplying two real numbers.

**Practice 6**
Use the definition of the quotient of two numbers to divide.

a. \( \frac{16}{-2} \)  
   b. \( 24 \div (-6) \)  
   c. \( \frac{-35}{-7} \)

Since the quotient \( a \div b \) can be written as the product \( a \cdot \frac{1}{b} \), it follows that sign patterns for dividing two real numbers are the same as sign patterns for multiplying two real numbers.

### Multiplying and Dividing Real Numbers

1. **The product or quotient of two numbers with the same sign is a positive number.**
2. **The product or quotient of two numbers with different signs is a negative number.**

**Example 7**
Divide.

a. \( \frac{-24}{-4} \)  
   b. \( \frac{-36}{3} \)  
   c. \( \frac{2}{3} \div \left( -\frac{5}{4} \right) \)  
   d. \( \frac{-3}{2} \div 9 \)

**Solution**

a. \( \frac{-24}{-4} = 6 \)  
   b. \( \frac{-36}{3} = -12 \)

\[ \frac{2}{3} \div \left( -\frac{5}{4} \right) = \frac{2}{3} \cdot \left( -\frac{4}{5} \right) = \frac{8}{15} \]

\[ \frac{-3}{2} \div 9 = \frac{-3}{2} \cdot \frac{1}{9} = \frac{-3 \cdot 1}{2 \cdot 3 \cdot 3} = \frac{-1}{6} \]

**Practice 7**
Divide.

a. \( \frac{-18}{-6} \)  
   b. \( \frac{-48}{3} \)  
   c. \( \frac{3}{5} \div \left( -\frac{1}{2} \right) \)  
   d. \( \frac{-4}{9} \div 8 \)

**Concept Check**

What is wrong with the following calculation?

\[ \frac{-36}{-9} \div 4 \]

The definition of the quotient of two real numbers does not allow for division by 0 because 0 does not have a multiplicative inverse. There is no number we can multiply 0 by to get 1. How then do we interpret \( \frac{3}{0} \)? We say that division by 0 is not allowed or not defined and that \( \frac{3}{0} \) does not represent a real number. The denominator of a fraction can never be 0.

Can the numerator of a fraction be 0? Can we divide 0 by a number? Yes. For example,

\[ \frac{0}{3} = 0 \cdot \frac{1}{3} = 0 \]

In general, the quotient of 0 and any nonzero number is 0.

**Zero as a Divisor or Dividend**

1. **The quotient of any nonzero real number and 0 is undefined.** In symbols, if \( a \neq 0 \), \( \frac{a}{0} \) is undefined.
2. **The quotient of 0 and any real number except 0 is 0.** In symbols, if \( a \neq 0 \), \( \frac{0}{a} = 0 \).
Section 1.7 Multiplying and Dividing Real Numbers

Notice that \( \frac{-12}{2} = -6, \frac{-12}{2} = -6 \), and \( \frac{-12}{2} = -6 \). This means that \( \frac{12}{-2} = \frac{-12}{2} = \frac{-12}{2} \).

In words, a single negative sign in a fraction can be written in the denominator, in the numerator, or in front of the fraction without changing the value of the fraction. Thus, \( \frac{1}{-7} = \frac{-1}{7} = \frac{-1}{7} \).

In general, if \( a \) and \( b \) are real numbers, \( b \neq 0 \), then \( \frac{a}{b} = \frac{-a}{-b} = \frac{-a}{b} \).

**Objective 4 Evaluating Expressions**

Examples combining basic arithmetic operations along with the principles of order of operations help us review these concepts.

**Example 9** Simplify each expression.

\[ \frac{(-12)(-3) + 3}{-7 - (-2)} \]

**Solution**

a. First, simplify the numerator and denominator separately, then divide.

\[ \frac{(-12)(-3) + 3}{-7 - (-2)} = \frac{36 + 3}{-7 + 2} = \frac{39}{-5} = -\frac{39}{5} \]

b. Simplify the numerator and denominator separately, then divide.

\[ \frac{2(-3)^2 - 20}{-5 + 4} = \frac{2 \cdot 9 - 20}{-5 + 4} = \frac{18 - 20}{-1} = \frac{-2}{-1} = 2 \]

**Practice 9** Simplify each expression.

a. \( \frac{(-8)(-11) - 4}{-9 - (-4)} \)

b. \( \frac{3(-2)^3 - 9}{-6 + 3} \)

Using what we have learned about multiplying and dividing real numbers, we continue to practice evaluating algebraic expressions.

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EXAMPLE 10 If \( x = -2 \) and \( y = -4 \), evaluate each expression.

a. \( 5x - y \)  

Solution  
a. Replace \( x \) with \( -2 \) and \( y \) with \( -4 \) and simplify.
\[
5x - y = 5(-2) - (-4) = -10 - (-4) = -10 + 4 = -6
\]

b. Replace \( x \) with \( -2 \) and \( y \) with \( -4 \).
\[
x^4 - y^2 = (-2)^4 - (-4)^2 = 16 - (16) = 0
\]

Substitute the given values for the variables.
Evaluate exponential expressions.
Subtract.

c. Replace \( x \) with \( -2 \) and \( y \) with \( -4 \) and simplify.
\[
\frac{3x}{2y} = \frac{3(-2)}{2(-4)} = \frac{-6}{-8} = \frac{3}{4}
\]

PRACTICE 10 If \( x = -5 \) and \( y = -2 \), evaluate each expression.

a. \( 7y - x \)  

b. \( x^2 - y^3 \)  

c. \( \frac{2x}{3y} \)

OBJECTIVE 5 Solving Applications That Involve Multiplying or Dividing Numbers

Many real-life problems involve multiplication and division of numbers.

EXAMPLE 11 Calculating a Total Golf Score

A professional golfer finished seven strokes under par \((-7)\) for each of three days of a tournament. What was her total score for the tournament?

Solution  
Although the key word is “total,” since this is repeated addition of the same number, we multiply.

In words: 
golfer’s total score = number of days \cdot score each day

\[\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]

Translate: 
golfer’s total = 3 \cdot (-7)

\[\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \]

\[= -21\]

Thus, the golfer’s total score was \(-21\), or 21 strokes under par.

PRACTICE 11 A card player had a score of \(-13\) for each of four games. Find the total score.

Graphing Calculator Explorations

Entering Negative Numbers on a Scientific Calculator

To enter a negative number on a scientific calculator, find a key marked \(\text{[+/-]}\). (On some calculators, this key is marked \(\text{CHS}\) for “change sign.”) To enter \(-8\), for example, press the keys \(8\ [+/-] \). The display will read \(-8\).
Entering Negative Numbers on a Graphing Calculator
To enter a negative number on a graphing calculator, find a key marked \(-\). Do not confuse this key with the key \([-\)], which is used for subtraction. To enter \(-8\), for example, press the keys \(\boxed{\text{-}}\) \(8\). The display will read \(-8\).

Operations with Real Numbers
To evaluate \(-2 \times (7 - 9) - 20\) on a calculator, press the keys
\[
\begin{align*}
2 & + \div \times (7 & - 9 & - 20) \\
& \text{or} \\
(\boxed{\text{-}}) & 2 & 7 & 9 & 1 & \boxed{\text{-}} & \boxed{\text{2}} & \boxed{0} & \boxed{\text{ENTER}}
\end{align*}
\]
The display will read \(-16\) or \(-2 \times (7 - 9) - 20 = -16\).

Use a calculator to simplify each expression.
1. \(-38(26 - 27)\)  
2. \(-59(-8) + 1726\)  
3. \(134 + 25(68 - 91)\)  
4. \(45(32) - 8(218)\)  
5. \(-50(294)\)  
6. \(-444 - 444.8\)  
7. \(9^2 - 4550\)  
8. \(5^9 - 6259\)  
9. \((-125)^2\) (Be careful.)  
10. \(-125^2\) (Be careful.)

Vocabulary, Readiness & Video Check
Use the choices below to fill in each blank. Some choices may be used more than once.
- positive
- 0
- negative
- undefined

1. If \(n\) is a real number, then \(n \cdot 0 = \) _________ and \(0 \cdot n = \) _________.
2. If \(n\) is a real number, but not 0, then \(\frac{0}{n} = \) _________ and we say \(\frac{n}{0}\) is _________.
3. The product of two negative numbers is a _________ number.
4. The quotient of two negative numbers is a _________ number.
5. The quotient of a positive number and a negative number is a _________ number.
6. The product of a positive number and a negative number is a _________ number.
7. The reciprocal of a positive number is a _________ number.
8. The opposite of a positive number is a _________ number.

Watch the section lecture video and answer the following questions.

9. Explain the significance of the use of parentheses when comparing Examples 6 and 7.
10. In Example 9, why is the reciprocal equal to \(\frac{3}{2}\) and not \(-\frac{3}{2}\)?
11. Before Example 11, the sign rules for division of real numbers are discussed. Are the sign rules for division the same as for multiplication? Why or why not?
12. In Example 17, the importance of placing the replacement values in parentheses when evaluating is emphasized. Why?
13. In Example 18, explain why each loss of 4 yards is represented by \(-4\) and not 4.
1.7  Exercise Set

Multiply. See Examples 1 through 3.

1. \(-6(4)\)  
2. \(-8(5)\)  
3. \((-1)\)  
4. \(7(-4)\)  
5. \(-5(-10)\)  
6. \(-6(-11)\)  
7. \(-3\cdot4\)  
8. \(-2\cdot8\)  
9. \(-7\cdot0\)  
10. \(-6\cdot0\)  
11. \(2(-9)\)  
12. \(1\cdot(5)\)  
13. \(-\frac{1}{2}\left(-\frac{3}{5}\right)\)  
14. \(-\frac{1}{8}\left(-\frac{1}{3}\right)\)  
15. \(-\frac{3}{4}\left(-\frac{8}{9}\right)\)  
16. \(-\frac{5}{6}\left(-\frac{3}{10}\right)\)  
17. \(5(-1.4)\)  
18. \(6(-2.5)\)  
19. \(-0.2(-0.7)\)  
20. \(-0.5(-0.3)\)  
21. \(-10(80)\)  
22. \(-20(60)\)  
23. \(4(-7)\)  
24. \(5(-9)\)  
25. \((-5)(-5)\)  
26. \((-7)(-7)\)  
27. \(\frac{2}{3}\left(-\frac{4}{9}\right)\)  
28. \(\frac{2}{7}\left(-\frac{2}{11}\right)\)  
29. \(-11(11)\)  
30. \(-12(12)\)  
31. \(-\frac{20}{25}\left(-\frac{5}{16}\right)\)  
32. \(-\frac{25}{36}\left(-\frac{6}{15}\right)\)  
33. \((-1)(-2)(-3)(-5)\)  
34. \((-2)(-3)(-4)(-2)\)  

Perform the indicated operations. See Example 2.

35. \((-2)(5) - (-11)(3)\)  
36. \(8(-3) - 4(-5)\)  
37. \((-6)(-1)(-2) - (-5)\)  
38. \(20 - (-4)(3)(-2)\)  

Divide. See Examples 6 through 8.

39. \(\frac{-9}{-2}\)  
40. \(\frac{20}{-10}\)  
41. \(\frac{-16}{-4}\)  
42. \(\frac{-18}{-6}\)  
43. \(\frac{-48}{-12}\)  
44. \(\frac{-60}{-5}\)  
45. \(\frac{0}{-4}\)  
46. \(\frac{5}{-9}\)  
47. \(\frac{-15}{3}\)  
48. \(\frac{24}{8}\)  
49. \(\frac{5}{0}\)  
50. \(\frac{3}{0}\)  
51. \(\frac{-12}{-4}\)  
52. \(\frac{-45}{-9}\)  
53. \(\frac{30}{2}\)  

MIXED PRACTICE

Simplify. See Examples 1 through 9.

54. \(\frac{-9(-3)}{-6}\)  
55. \(\frac{12}{9 - 12}\)  
56. \(\frac{-6^2 + 4}{-2}\)  
57. \(\frac{8 + (-4)^2}{4 - 12}\)  
58. \(\frac{22 + (3)(-2)}{-5 - 2}\)  
59. \(\frac{16}{2(-7)}\)  
60. \(\frac{-6 - 2(-3)}{4 - 3(-2)}\)  
61. \(\frac{-3 - 2(-9)}{-15 - 3(-4)}\)  
62. \(\frac{[5 - 9] + [10 - 15]}{|2(-3)| - [-2\cdot2]}\)

If \(x = -5\) and \(y = -3\), evaluate each expression. See Example 10.

63. \(3x + 2y\)  
64. \(4x + 5y\)  
65. \(x^2 - 2y^2\)  
66. \(y^3 + 3x\)  
67. \(2y - 12\)  
68. \(x - 4\)  
69. \(\frac{2x - 5}{y - 2}\)  
70. \(\frac{4 - 2x}{y + 3}\)  
71. \(\frac{-3 - y}{x - 4}\)  
72. \(\frac{3}{0}\)  
73. \(\frac{3}{0}\)  
74. \(\frac{3}{0}\)  
75. \(\frac{3}{0}\)  
76. \(\frac{3}{0}\)  
77. \(\frac{3}{0}\)  
78. \(\frac{3}{0}\)  
79. \(\frac{3}{0}\)  
80. \(\frac{3}{0}\)  
81. \(\frac{3}{0}\)  
82. \(\frac{3}{0}\)  
83. \(\frac{3}{0}\)  
84. \(\frac{3}{0}\)  
85. \(\frac{3}{0}\)  
86. \(\frac{3}{0}\)  
87. \(\frac{3}{0}\)  
88. \(\frac{3}{0}\)  
89. \(\frac{3}{0}\)  
90. \(\frac{3}{0}\)  
91. \(\frac{3}{0}\)  
92. \(\frac{3}{0}\)  
93. \(\frac{3}{0}\)  
94. \(\frac{3}{0}\)  
95. \(\frac{3}{0}\)  
96. \(\frac{3}{0}\)  
97. \(\frac{3}{0}\)  
98. \(\frac{3}{0}\)  
99. \(\frac{3}{0}\)  
100. \(\frac{3}{0}\)  
101. \(\frac{3}{0}\)  
102. \(\frac{3}{0}\)  

Find each reciprocal or multiplicative inverse. See Example 5.

51. \(9\)  
52. \(100\)  
53. \(\frac{2}{3}\)  
54. \(\frac{1}{7}\)  
55. \(-14\)  
56. \(-8\)  
57. \(-\frac{3}{11}\)  
58. \(-\frac{6}{13}\)  
59. \(0.2\)  
60. \(1.5\)  
61. \(-\frac{1}{6.5}\)  
62. \(-\frac{1}{8.9}\)

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Section 1.7  Multiplying and Dividing Real Numbers

123. A deep-sea diver must move up or down in the water in short steps to keep from getting a physical condition called the “bends.” Suppose a diver moves down from the surface in five steps of 20 feet each. Represent his total movement as a product of signed numbers and find the total depth.

124. A weather forecaster predicts that the temperature will drop five degrees each hour for the next six hours. Represent this drop as a product of signed numbers and find the total drop in temperature.

Decide whether the given number is a solution of the given equation.

125. Is 7 a solution of \(-5x = -35\)?

126. Is \(-4\) a solution of \(2x = x - 1\)?

127. Is \(-3\) a solution of \(\frac{45}{x} = -15\)?

128. Is 5 a solution of \(-3x - 5 = -20\)?

129. Is \(-4\) a solution of \(2x + 4 = x + 8\)?

TRANSLATING

Translate each phrase into an expression. Use \(x\) to represent “a number.” See Example 11.

113. The product of \(-71\) and a number

114. The quotient of \(-8\) and a number

115. Subtract a number from \(-16\).

116. The sum of a number and \(-12\)

117. \(-29\) increased by a number

118. The difference of a number and \(-10\)

119. Divide a number by \(-33\).

120. Multiply a number by \(-17\).

Solve. See Example 11.

121. A football team lost four yards on each of three consecutive plays. Represent the total loss as a product of signed numbers and find the total loss.

122. Joe Norstrom lost \$400\) on each of seven consecutive days in the stock market. Represent his total loss as a product of signed numbers and find his total loss.

CONCEPT EXTENSIONS

Study the bar graph below showing the average surface temperatures of planets. Use Exercises 131 and 132 to complete the planet temperatures on the graph. (Pluto is now classified as a dwarf planet.)

131. The surface temperature of Jupiter is twice the temperature of Mars. Find this temperature.

132. The surface temperature of Neptune is equal to the temperature of Mercury divided by \(-1\). Find this temperature.

133. Explain why the product of an even number of negative numbers is a positive number.

134. If \(a\) and \(b\) are any real numbers, is the statement \(a \cdot b = b \cdot a\) always true? Why or why not?

135. Find any real numbers that are their own reciprocal.

136. Explain why 0 has no reciprocal.

If \(q\) is a negative number, \(r\) is a negative number, and \(t\) is a positive number, determine whether each expression simplifies to a positive or negative number. If it is not possible to determine, state so.

137. \(\frac{q}{r \cdot t}\)

138. \(q^2 \cdot r \cdot t\)

139. \(q + t\)

140. \(t + r\)

141. \(t(q + r)\)

142. \(r(q - t)\)

Write each of the following as an expression and evaluate.

143. The sum of \(-2\) and the quotient of \(-15\) and 3

144. The sum of 1 and the product of \(-8\) and \(-5\)

145. Twice the sum of \(-5\) and \(-3\)

146. 7 subtracted from the quotient of 0 and 5
1.8 Properties of Real Numbers

**OBJECTIVES**

1. Use the Commutative and Associative Properties.
2. Use the Distributive Property.
3. Use the Identity and Inverse Properties.

**OBJECTIVE 1** Using the Commutative and Associative Properties

In this section, we give names to properties of real numbers with which we are already familiar. Throughout this section, the variables \(a\), \(b\), and \(c\) represent real numbers.

We know that order does not matter when adding numbers. For example, we know that \(7 + 5\) is the same as \(5 + 7\). This property is given a special name—the commutative property of addition.

We also know that order does not matter when multiplying numbers. For example, we know that \(-5 \cdot 2 = \frac{5}{-2}\). This property means that multiplication is commutative also and is called the commutative property of multiplication.

**Commutative Properties**

**Addition:** \(a + b = b + a\)

**Multiplication:** \(a \cdot b = b \cdot a\)

These properties state that the order in which any two real numbers are added or multiplied does not change their sum or product. For example, if we let \(a = 3\) and \(b = 5\), then the commutative properties guarantee that

\[3 + 5 = 5 + 3\]

and

\[3 \cdot 5 = 5 \cdot 3\]

**Helpful Hint**

Is subtraction also commutative? Try an example. Does \(3 - 2 = 2 - 3\)? No! The left side of this statement equals 1; the right side equals -1. There is no commutative property of subtraction. Similarly, there is no commutative property for division. For example, \(10 ÷ 2\) does not equal \(2 ÷ 10\).

**EXAMPLE 1** Use a commutative property to complete each statement.

**a.** \(x + 5 = \square\)

**b.** \(3 \cdot x = \square\)

**Solution**

**a.** \(x + 5 = 5 + x\) By the commutative property of addition

**b.** \(3 \cdot x = x \cdot 3\) By the commutative property of multiplication

**PRACTICE 1** Use a commutative property to complete each statement.

**a.** \(x \cdot 8 = \square\)

**b.** \(x + 17 = \square\)

**CONCEPT CHECK**

Which of the following pairs of actions are commutative?

**a.** “raking the leaves” and “bagging the leaves”

**b.** “putting on your left glove” and “putting on your right glove”

**c.** “putting on your coat” and “putting on your shirt”

**d.** “reading a novel” and “reading a newspaper”

Answers to Concept Check:  

b, d

Let’s now discuss grouping numbers. We know that when we add three numbers, the way in which they are grouped or associated does not change their sum. For example, we know that \(2 + (3 + 4) = 2 + 7 = 9\). This result is the same if we group the numbers differently. In other words, \((2 + 3) + 4 = 5 + 4 = 9\) also. Thus, \(2 + (3 + 4) = (2 + 3) + 4\). This property is called the associative property of addition.

We also know that changing the grouping of numbers when multiplying does not change their product. For example, \(2 \cdot (3 \cdot 4) = (2 \cdot 3) \cdot 4\) (check it). This is the associative property of multiplication.
Section 1.8 Properties of Real Numbers

These properties state that the way in which three numbers are grouped does not change their sum or their product.

**EXAMPLE 2** Use an associative property to complete each statement.

**a.** $5 + (4 + 6) = \quad$ 
**b.** $(-1) \cdot 2 \cdot 5 = \quad$

**Solution**

**a.** $5 + (4 + 6) = (5 + 4) + 6 \quad$ By the associative property of addition

**b.** $(-1) \cdot 2 \cdot 5 = -1 \cdot (2 \cdot 5) \quad$ By the associative property of multiplication

**PRACTICE**

Use an associative property to complete each statement.

**a.** $(2 + 9) + 7 = \quad$

**b.** $-4 \cdot (2 \cdot 7) = \quad$

**Helpful Hint**

Remember the difference between the commutative properties and the associative properties. The commutative properties have to do with the order of numbers, and the associative properties have to do with the grouping of numbers.

Let's now illustrate how these properties can help us simplify expressions.

**EXAMPLE 3** Simplify each expression.

**a.** $10 + (x + 12)$  
**b.** $-3(7x)$

**Solution**

**a.** $10 + (x + 12) = 10 + (12 + x) \quad$ By the commutative property of addition

$= (10 + 12) + x \quad$ By the associative property of addition

$= 22 + x \quad$ Add.

**b.** $-3(7x) = (-3 \cdot 7)x \quad$ By the associative property of multiplication

$= -21x \quad$ Multiply.

**PRACTICE**

Simplify each expression.

**a.** $(5 + x) + 9$  
**b.** $5(-6x)$

**OBJECTIVE**

Using the Distributive Property

The **distributive property of multiplication over addition** is used repeatedly throughout algebra. It is useful because it allows us to write a product as a sum or a sum as a product.

We know that $7(2 + 4) = 7(6) = 42$. Compare that with $7(2) + 7(4) = 14 + 28 = 42$. Since both original expressions equal 42, they must equal each other, or

$7(2 + 4) = 7(2) + 7(4)$

This is an example of the distributive property. The product on the left side of the equal sign is equal to the sum on the right side. We can think of the 7 as being distributed to each number inside the parentheses.

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Chapter 1  
Review of Real Numbers

Since multiplication is commutative, this property can also be written as

\[(b + c)a = ba + ca\]

The distributive property can also be extended to more than two numbers inside the parentheses. For example,

\[3(x + y + z) = 3(x) + 3(y) + 3(z)\]
\[= 3x + 3y + 3z\]

Since we define subtraction in terms of addition, the distributive property is also true for subtraction. For example,

\[2(x - y) = 2(x) - 2(y)\]
\[= 2x - 2y\]

**Example 4**  
Use the distributive property to write each expression without parentheses.

- **a.** \(2(x + y)\)  
- **b.** \(-5(-3 + 2z)\)  
- **c.** \(5(x + 3y - z)\)  
- **d.** \(-1(2 - y)\)  
- **e.** \(-(3 + x - w)\)  
- **f.** \(\frac{1}{2}(6x + 14) + 10\)

**Solution**

- **a.** \(2(x + y) = 2 \cdot x + 2 \cdot y\)
  \[= 2x + 2y\]

- **b.** \(-5(-3 + 2z) = -5(-3) + (-5)(2z)\)
  \[= 15 - 10z\]

- **c.** \(5(x + 3y - z) = 5(x) + 5(3y) - 5(z)\)
  \[= 5x + 15y - 5z\]

- **d.** \(-1(2 - y) = (-1)(2) - (-1)(y)\)
  \[= -2 + y\]

- **e.** \(-(3 + x - w) = -1(3 + x - w)\)
  \[= (-1)(3) + (-1)(x) - (-1)(w)\]
  \[= -3 - x + w\]

- **f.** \(\frac{1}{2}(6x + 14) + 10 = \frac{1}{2}(6x) + \frac{1}{2}(14) + 10\)
  Apply the distributive property.
  \[= 3x + 7 + 10\]
  Multiply.
  \[= 3x + 17\]
  Add.
Section 1.8 Properties of Real Numbers

Notice that 0 is the only number that can be added to any real number with the result that the sum is the same real number. Also, 1 is the only number that can be multiplied by any real number with the result that the product is the same real number.

Additive inverses or opposites were introduced in Section 1.5. Two numbers are called additive inverses or opposites if their sum is 0. The additive inverse or opposite of 6 is \(-6\) because \(6 + (-6) = 0\). The additive inverse or opposite of \(-5\) is 5 because \(-5 + 5 = 0\).

### Practice

Use the distributive property to write each expression without parentheses. Then simplify if possible.

- a. \(5(x - y)\)
- b. \(-6(4 + 2t)\)
- c. \(2(3x - 4y - z)\)
- d. \((3 - y) \cdot (-1)\)
- e. \(-(x - 7 + 2s)\)
- f. \(\frac{1}{2}(2x + 4) + 9\)

We can use the distributive property in reverse to write a sum as a product.

### Example 5

Use the distributive property to write each sum as a product.

- a. \(8 \cdot 2 + 8 \cdot x\)
- b. \(7s + 7t\)

**Solution**

- a. \(8 \cdot 2 + 8 \cdot x = 8(2 + x)\)
- b. \(7s + 7t = 7(s + t)\)

### Practice

Use the distributive property to write each sum as a product.

- a. \(5 \cdot w + 5 \cdot 3\)
- b. \(9w + 9z\)

### Objective

Using the Identity and Inverse Properties

Next, we look at the identity properties. The number 0 is called the identity for addition because when 0 is added to any real number, the result is the same real number. In other words, the identity of the real number is not changed.

The number 1 is the identity element for multiplication because when a real number is multiplied by 1, the result is the same real number. In other words, the identity of the real number is not changed.

<table>
<thead>
<tr>
<th>Identities for Addition and Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 is the identity element for addition.</td>
</tr>
<tr>
<td>(a + 0 = a) and (0 + a = a)</td>
</tr>
<tr>
<td>1 is the identity element for multiplication.</td>
</tr>
<tr>
<td>(a \cdot 1 = a) and (1 \cdot a = a)</td>
</tr>
</tbody>
</table>

Notice that 0 is the only number that can be added to any real number with the result that the sum is the same real number. Also, 1 is the only number that can be multiplied by any real number with the result that the product is the same real number.

Additive inverses or opposites were introduced in Section 1.5. Two numbers are called additive inverses or opposites if their sum is 0. The additive inverse or opposite of 6 is \(-6\) because \(6 + (-6) = 0\). The additive inverse or opposite of \(-5\) is 5 because \(-5 + 5 = 0\).
Reciprocals or multiplicative inverses were introduced in Section 1.3. Two non-zero numbers are called reciprocals or multiplicative inverses if their product is 1. The reciprocal or multiplicative inverse of \( \frac{2}{3} \) is \( \frac{3}{2} \) because \( \frac{2}{3} \cdot \frac{3}{2} = 1 \). Likewise, the reciprocal of \(-5\) is \(-\frac{1}{5}\) because \(-5 \cdot \left(-\frac{1}{5}\right) = 1\).

\[ \text{CONCEPT CHECK} \]

Which of the following, 1, \( \frac{3}{10} \), \( -\frac{3}{10} \), 0, \( -\frac{3}{10} \), \( -\frac{3}{10} \), is the

a. opposite of \( \frac{3}{10} \)?

b. reciprocal of \( \frac{3}{10} \)?

Additive or Multiplicative Inverses

The numbers \( a \) and \( -a \) are additive inverses or opposites of each other because their sum is 0; that is,

\[ a + (-a) = 0 \]

The numbers \( b \) and \( \frac{1}{b} \) (for \( b \neq 0 \)) are reciprocals or multiplicative inverses of each other because their product is 1; that is,

\[ b \cdot \frac{1}{b} = 1 \]

**EXAMPLE 6** Name the property or properties illustrated by each true statement.

**Solution**

a. \( 3 \cdot y = y \cdot 3 \)  
   Commutative property of multiplication  
   (order changed)

b. \( (x + 7) + 9 = x + (7 + 9) \)  
   Associative property of addition  
   (grouping changed)

c. \( (b + 0) + 3 = b + 3 \)  
   Identity element for addition

d. \( 0.2 \cdot (z \cdot 5) = 0.2 \cdot (5 \cdot z) \)  
   Commutative property of multiplication  
   (order changed)

\[ e. -2 \cdot \left(-\frac{1}{2}\right) = 1 \]

\[ f. -2 + 2 = 0 \]

\[ g. -6 \cdot (y \cdot 2) = (-6 \cdot 2) \cdot y \]

Additive inverse property

Commutative and associative properties of multiplication (order and grouping changed)

**PRACTICE 6** Name the property or properties illustrated by each true statement.

a. \( (7 \cdot 3x) \cdot 4 = (3x \cdot 7) \cdot 4 \)  
   Commutative property of multiplication

b. \( 6 + (3 + y) = (6 + 3) + y \)  
   Associative property of addition

\[ c. 8 + (t + 0) = 8 + t \]

Identity element for addition

\[ d. -\frac{3}{4} \cdot \left(-\frac{4}{3}\right) = 1 \]

Multiplicative inverse property

\[ e. (2 + x) + 5 = 5 + (2 + x) \]

Commutative property of addition

\[ f. 3 + (-3) = 0 \]

Additive inverse property

\[ g. (-3b) \cdot 7 = (-3 \cdot 7) \cdot b \]

Commutative and associative properties of multiplication

Answers to Concept Check:

a. \( \frac{3}{10} \)

b. \( \frac{10}{3} \)

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Section 1.8
Properties of Real Numbers

Use the choices below to fill in each blank.
- distributive property
- associative property of multiplication
- opposites or additive inverses
- commutative property of addition
- reciprocals or multiplicative inverses
- associative property of addition

1. \( x + 5 = 5 + x \) is a true statement by the ________.
2. \( x \cdot 5 = 5 \cdot x \) is a true statement by the ________.
3. \( 3(y + 6) = 3 \cdot y + 3 \cdot 6 \) is true by the ________.
4. \( 2 \cdot (x \cdot y) = (2 \cdot x) \cdot y \) is a true statement by the ________.
5. \( x + (7 + y) = (x + 7) + y \) is a true statement by the ________.
6. The numbers \(-\frac{2}{3} \) and \( -\frac{3}{2} \) are called ________.
7. The numbers \( \frac{2}{3} \) and \( \frac{2}{3} \) are called ________.

8. The commutative properties are discussed in Examples 1 and 2 and the associative properties are discussed in Examples 3–7. What’s the one word used again and again to describe the commutative property? The associative property?

9. In Example 10, what point is made about the term 2?

10. Complete these statements based on the lecture given before Example 12.
   - The identity element for addition is ________ because if we add ________ to any real number, the result is that real number.
   - The identity element for multiplication is ________ because any real number times ________ gives a result of that original real number.

Use a commutative property to complete each statement. See Example 1.

1. \( x + 16 = \) ________
2. \( 4 + y = \) ________
3. \( -4 \cdot y = \) ________
4. \( -2 \cdot x = \) ________
5. \( xy = \) ________
6. \( ab = \) ________
7. \( 2x + 13 = \) ________
8. \( 19 + 3y = \) ________

Use an associative property to complete each statement. See Example 2.

9. \( (xy) \cdot z = \) ________
10. \( 3 \cdot (xy) = \) ________
11. \( 2 + (a + b) = \) ________
12. \( (y + 4) + z = \) ________
13. \( 4 \cdot (ab) = \) ________
14. \( (-3y) \cdot z = \) ________
15. \( (a + b) + c = \) ________
16. \( 6 + (r + s) = \) ________

Use the commutative and associative properties to simplify each expression. See Example 3.

17. \( 8 + (9 + b) \)
18. \( (r + 3) + 11 \)
19. \( 4(6y) \)
20. \( 2(42x) \)
21. \( \frac{1}{5} (5y) \)
22. \( \frac{1}{8} (8z) \)
23. \( (13 + a) + 13 \)
24. \( 7 + (x + 4) \)
25. \( -9(8x) \)
26. \( -3(12y) \)
27. \( \frac{3}{4} \cdot \frac{4}{3} = \) ________
28. \( \frac{2}{7} + \frac{7}{2} = \) ________
29. \( \frac{2}{3} + \left( \frac{4}{3} + x \right) \)
30. \( \frac{7}{9} + \left( \frac{2}{9} + y \right) \)

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Use the distributive property to write each expression without parentheses. Then simplify the result. See Example 4.

31. \(4(x + y)\)  
32. \(7(a + b)\)  
33. \(9(x - 6)\)  
34. \(11(y - 4)\)  
35. \(2(3x + 5)\)  
36. \(5(7 + 8y)\)  
37. \(7(4x - 3)\)  
38. \(3(8x - 1)\)  
39. \(3(6 + x)\)  
40. \(2(x + 5)\)  
41. \(-2(y - z)\)  
42. \(-3(z - y)\)  
\(\text{Concept Check in this section.}\)

Use the distributive property to write each sum as a product. See Example 5.

63. \(4 \cdot 1 + 4 \cdot y\)  
64. \(14 \cdot z + 14 \cdot 5\)  
65. \(11x + 11y\)  
66. \(9a + 9b\)  
67. \((-1) \cdot 5 + (-1) \cdot x\)  
68. \((-3)a + (-3)b\)  
69. \(30a + 30b\)  
70. \(25x + 25y\)

Name the properties illustrated by each true statement. See Example 6.

71. \(3 \cdot 5 = 5 \cdot 3\)  
72. \(4(3 + 8) = 4 \cdot 3 + 4 \cdot 8\)  
73. \(2 + (x + 5) = (2 + x) + 5\)  
74. \((x + 9) + 3 = (9 + x) + 3\)  
75. \(9(3 + 7) = 9 \cdot 3 + 9 \cdot 7\)  
76. \(1 \cdot 9 = 9\)  
77. \((4 \cdot y) \cdot 9 = 4 \cdot (y \cdot 9)\)  
78. \(6 \cdot \frac{1}{6} = 1\)

\(\text{CONCEPT EXTENSIONS}\)

Fill in the table with the opposite (additive inverse) and the reciprocal (multiplicative inverse). Assume that the value of each expression is not 0.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Opposite</th>
<th>Reciprocal</th>
</tr>
</thead>
<tbody>
<tr>
<td>85. (8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>86. (-\frac{2}{3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>87. (x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>88. (4y)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>89. (\frac{1}{2x})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90. (7x)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Decide whether each statement is true or false. See the second Concept Check in this section.

91. The opposite of \(-\frac{a}{2}\) is \(-\frac{2}{a}\).
92. The reciprocal of \(-\frac{a}{2}\) is \(-\frac{a}{2}\).
102. a. \((x + y) + z = x + (y + z)\)
   b. \((y + z) + x\)
   c. \((z + y) + x\)

103. Explain why 0 is called the identity element for addition.

104. Explain why 1 is called the identity element for multiplication.

105. Write an example that shows that division is not commutative.

106. Write an example that shows that subtraction is not commutative.

---

**Chapter 1 Vocabulary Check**

Fill in each blank with one of the words or phrases listed below.

- set
- inequality symbols
- opposites
- absolute value
- numerator
- denominator
- grouping symbols
- exponent
- base
- reciprocals
- variable
- equation
- solution

1. The symbols \(\neq, <, \text{ and } >\) are called ____________.
2. A mathematical statement that two expressions are equal is called a(n) ____________.
3. The ____________ of a number is the distance between that number and 0 on a number line.
4. A symbol used to represent a number is called a(n) ____________.
5. Two numbers that are the same distance from 0 but lie on opposite sides of 0 are called ____________.
6. The number in a fraction above the fraction bar is called the ____________.
7. A(n) ____________ of an equation is a value for the variable that makes the equation a true statement.
8. Two numbers whose product is 1 are called ____________.
9. In \(2^3\), the 2 is called the ____________ and the 3 is called the ____________.
10. The number in a fraction below the fraction bar is called the ____________.
11. Parentheses and brackets are examples of ____________.
12. A(n) ____________ is a collection of objects.

---

**Chapter 1 Highlights**

**DEFINITIONS AND CONCEPTS**

Section 1.2  Symbols and Sets of Numbers

- **A set** is a collection of objects, called elements, enclosed in braces.

- **Natural Numbers**: \(\{1, 2, 3, 4, \ldots\}\)
- **Whole Numbers**: \(\{0, 1, 2, 3, 4, \ldots\}\)
- **Integers**: \(\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}\)
- **Rational Numbers**: [real numbers that can be expressed as a quotient of integers]
- **Irrational Numbers**: [real numbers that cannot be expressed as a quotient of integers]
- **Real Numbers**: [all numbers that correspond to a point on the number line]

**EXAMPLES**

- Given the set \(\{-3.4, \sqrt{3}, 0, \frac{2}{3}, 5, -4\}\), list the numbers that belong to the set of
  - Natural numbers: 5
  - Whole numbers: 0, 5
  - Integers: \(-4, 0, 5\)
  - Rational numbers: \(-4, -3.4, 0, \frac{2}{3}, 5\)
  - Irrational Numbers: \(\sqrt{3}\)
  - Real numbers: \(-4, -3.4, 0, \frac{2}{3}, \sqrt{3}, 5\)

(continued)
Section 1.2 Symbols and Sets of Numbers (continued)

A line used to picture numbers is called a **number line**. The absolute value of a real number \(a\), denoted by \(|a|\), is the distance between \(a\) and 0 on a number line.

**Symbols:**
- \(=\) is equal to
- \(\neq\) is not equal to
- \(>\) is greater than
- \(<\) is less than
- \(\leq\) is less than or equal to
- \(\geq\) is greater than or equal to

**Order Property for Real Numbers**

For any two real numbers \(a\) and \(b\), \(a\) is less than \(b\) if \(a\) is to the left of \(b\) on a number line.

Section 1.3 Fractions and Mixed Numbers

A quotient of two integers is called a **fraction**. The **numerator** of a fraction is the top number. The **denominator** of a fraction is the bottom number.

If \(a \cdot b = c\), then \(a\) and \(b\) are **factors** and \(c\) is the **product**.

A fraction is in **lowest terms** or **simplest form** when the numerator and the denominator have no factors in common other than 1.

**To write a fraction in simplest form**, factor the numerator and the denominator; then apply the fundamental principle.

Two fractions are **reciprocals** if their product is 1.

The reciprocal of \(\frac{a}{b}\) is \(\frac{b}{a}\).

**To multiply fractions**, numerator times numerator is the numerator of the product and denominator times denominator is the denominator of the product.

**To divide fractions**, multiply the first fraction by the reciprocal of the second fraction.

**To add fractions with the same denominator**, add the numerators and place the sum over the common denominator.

**To subtract fractions with the same denominator**, subtract the numerators and place the difference over the common denominator.

Fractions that represent the same quantity are called **equivalent fractions**.

### Examples

<table>
<thead>
<tr>
<th>DEFINITIONS AND CONCEPTS</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A line used to picture numbers is called a number line.</strong></td>
<td></td>
</tr>
<tr>
<td>The <strong>absolute value</strong> of a real number (a), denoted by (</td>
<td>a</td>
</tr>
<tr>
<td><strong>Symbols:</strong> (=) is equal to (\neq) is not equal to (&gt;) is greater than (&lt;) is less than (\leq) is less than or equal to (\geq) is greater than or equal to</td>
<td></td>
</tr>
<tr>
<td><strong>Order Property for Real Numbers</strong></td>
<td></td>
</tr>
<tr>
<td>For any two real numbers (a) and (b), (a) is less than (b) if (a) is to the left of (b) on a number line.</td>
<td></td>
</tr>
<tr>
<td><strong>A quotient of two integers is called a fraction.</strong></td>
<td></td>
</tr>
<tr>
<td>The <strong>numerator</strong> of a fraction is the top number. The <strong>denominator</strong> of a fraction is the bottom number.</td>
<td></td>
</tr>
<tr>
<td>If (a \cdot b = c), then (a) and (b) are <strong>factors</strong> and (c) is the <strong>product</strong>.</td>
<td></td>
</tr>
<tr>
<td>A fraction is in <strong>lowest terms</strong> or <strong>simplest form</strong> when the numerator and the denominator have no factors in common other than 1.</td>
<td></td>
</tr>
<tr>
<td><strong>To write a fraction in simplest form</strong>, factor the numerator and the denominator; then apply the fundamental principle.</td>
<td></td>
</tr>
<tr>
<td>Two fractions are <strong>reciprocals</strong> if their product is 1.</td>
<td></td>
</tr>
<tr>
<td>The reciprocal of (\frac{a}{b}) is (\frac{b}{a}).</td>
<td></td>
</tr>
<tr>
<td><strong>To multiply fractions</strong>, numerator times numerator is the numerator of the product and denominator times denominator is the denominator of the product.</td>
<td></td>
</tr>
<tr>
<td><strong>To divide fractions</strong>, multiply the first fraction by the reciprocal of the second fraction.</td>
<td></td>
</tr>
<tr>
<td><strong>To add fractions with the same denominator</strong>, add the numerators and place the sum over the common denominator.</td>
<td></td>
</tr>
<tr>
<td><strong>To subtract fractions with the same denominator</strong>, subtract the numerators and place the difference over the common denominator.</td>
<td></td>
</tr>
<tr>
<td>Fractions that represent the same quantity are called <strong>equivalent fractions</strong>.</td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{Symbols: } & = \text{ is equal to} \\
& \neq \text{ is not equal to} \\
& > \text{ is greater than} \\
& < \text{ is less than} \\
& \leq \text{ is less than or equal to} \\
& \geq \text{ is greater than or equal to} \\
\text{Order Property for Real Numbers: } & \text{For any two real numbers } a \text{ and } b, a \text{ is less than } b \text{ if } a \text{ is to the left of } b \text{ on a number line.} \\
\text{A quotient of two integers is called a } \text{fraction.} \\
& \text{The numerator of a fraction is the top number.} \\
& \text{The denominator of a fraction is the bottom number.} \\
\text{If } a \cdot b = c, \text{ then } a \text{ and } b \text{ are factors and } c \text{ is the product.} \\
\text{A fraction is in lowest terms or simplest form when the numerator and the denominator have no factors in common other than 1.} \\
\text{To write a fraction in simplest form, factor the numerator and the denominator; then apply the fundamental principle.} \\
\text{Two fractions are reciprocals if their product is 1.} \\
\text{The reciprocal of } \frac{a}{b} \text{ is } \frac{b}{a}. \\
\text{To multiply fractions, numerator times numerator is the numerator of the product and denominator times denominator is the denominator of the product.} \\
\text{To divide fractions, multiply the first fraction by the reciprocal of the second fraction.} \\
\text{To add fractions with the same denominator, add the numerators and place the sum over the common denominator.} \\
\text{To subtract fractions with the same denominator, subtract the numerators and place the difference over the common denominator.} \\
\text{Fractions that represent the same quantity are called equivalent fractions.}
\end{align*}
\]
### Section 1.4  Exponents, Order of Operations, Variable Expressions, and Equations

**The expression** $a^n$ is an **exponential expression**. The number $a$ is called the **base**; it is the repeated factor. The number $n$ is called the **exponent**; it is the number of times that the base is a factor.

**Order of Operations**

Simplify expressions in the following order. If grouping symbols are present, simplify expressions within those first, starting with the innermost set. Also, simplify the numerator and the denominator of a fraction separately.

1. Simplify exponential expressions.
2. Multiply or divide in order from left to right.
3. Add or subtract in order from left to right.

A symbol used to represent a number is called a **variable**.

An **algebraic expression** is a collection of numbers, variables, operation symbols, and grouping symbols.

**To evaluate an algebraic expression** containing a variable, substitute a given number for the variable and simplify.

A mathematical statement that two expressions are equal is called an **equation**.

A **solution** of an equation is a value for the variable that makes the equation a true statement.

#### Examples

<table>
<thead>
<tr>
<th>Expression</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4^2 = 4 \cdot 4 \cdot 4$</td>
<td>$64$</td>
</tr>
<tr>
<td>$7^2 = 7 \cdot 7$</td>
<td>$49$</td>
</tr>
<tr>
<td>$\frac{8^2 + 5(7 - 3)}{3 \cdot 7} = \frac{8^2 + 5(4)}{21}$</td>
<td>$\frac{64 + 20}{21} = \frac{84}{21} = 4$</td>
</tr>
</tbody>
</table>

Examples of variables are:

$q, x, z$

Examples of algebraic expressions are:

$5x, 2(y - 6), \frac{q^2 - 3q + 1}{6}$

Evaluate $x^2 - y^2$ if $x = 5$ and $y = 3$.

$x^2 - y^2 = (5)^2 - (3)^2 = 25 - 9 = 16$

Examples of equations are:

$3x - 9 = 20$

$A = \pi r^2$

Determine whether $4$ is a solution of $5x + 7 = 27$.

$5x + 7 = 27$

$5(4) + 7 \overset{?}{=} 27$

$20 + 7 \overset{?}{=} 27$

$27 = 27$  **True**

4 is a solution.

#### Section 1.5  Adding Real Numbers

**To Add Two Numbers with the Same Sign**

1. Add their absolute values.
2. Use their common sign as the sign of the sum.

**To Add Two Numbers with Different Signs**

1. Subtract their absolute values.
2. Use the sign of the number whose absolute value is larger as the sign of the sum.

Add.

$10 + 7 = 17$

$-3 + (-8) = -11$

$-25 + 5 = -20$

$14 + (-9) = 5$

(continued)
### Chapter 1 Review of Real Numbers

#### Section 1.5 Adding Real Numbers (continued)

<table>
<thead>
<tr>
<th><strong>DEFINITIONS AND CONCEPTS</strong></th>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Two numbers that are the same distance from 0 but lie on opposite sides of 0 are called <strong>opposites</strong> or <strong>additive inverses</strong>. The opposite of a number ( a ) is denoted by (-a).</td>
<td>The opposite of (-7) is (7). The opposite of (123) is (-123).</td>
</tr>
<tr>
<td>The sum of a number ( a ) and its opposite, (-a), is 0. ( a + (-a) = 0 )</td>
<td>(-4 + 4 = 0) (12 + (-12) = 0) (-(-8) = 8) (-(-14) = 14)</td>
</tr>
<tr>
<td>If ( a ) is a number, then (-a) is the opposite of (a).</td>
<td></td>
</tr>
</tbody>
</table>

#### Section 1.6 Subtracting Real Numbers

To subtract two numbers \(a\) and \(b\), add the first number \(a\) to the opposite of the second number \(-b\). 

\[ a - b = a + (-b) \]

<table>
<thead>
<tr>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtract.</td>
</tr>
<tr>
<td>(3 - (-44) = 3 + 44 = 47)</td>
</tr>
<tr>
<td>(-5 - 22 = -5 + (-22) = -27)</td>
</tr>
<tr>
<td>(-30 - (-30) = -30 + 30 = 0)</td>
</tr>
</tbody>
</table>

#### Section 1.7 Multiplying and Dividing Real Numbers

**Quotient of two real numbers** 

\[
\frac{a}{b} = a \cdot \frac{1}{b}
\]

**Multiplying and Dividing Real Numbers**

The product or quotient of two numbers with the same sign is a positive number. The product or quotient of two numbers with different signs is a negative number.

<table>
<thead>
<tr>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply or divide.</td>
</tr>
<tr>
<td>(7 \cdot 8 = 56) (-7 \cdot (-8) = 56)</td>
</tr>
<tr>
<td>(-2 \cdot 4 = -8) (2 \cdot (-4) = -8)</td>
</tr>
<tr>
<td>(\frac{90}{10} = 9) (-\frac{90}{10} = -9)</td>
</tr>
<tr>
<td>(\frac{42}{-6} = -7) (-\frac{42}{6} = -7)</td>
</tr>
</tbody>
</table>

**Products and Quotients Involving Zero**

The product of 0 and any number is 0. \(b \cdot 0 = 0\) and \(0 \cdot b = 0\)

The quotient of a nonzero number and 0 is undefined. \(\frac{b}{0}\) is undefined.

The quotient of 0 and any nonzero number is 0. \(\frac{0}{b} = 0\)

<table>
<thead>
<tr>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply or divide.</td>
</tr>
<tr>
<td>(-4 \cdot 0 = 0) (0 \left(\frac{-3}{4}\right) = 0)</td>
</tr>
<tr>
<td>(\frac{-85}{0}) is undefined.</td>
</tr>
<tr>
<td>(\frac{0}{18} = 0) (\frac{0}{-47} = 0)</td>
</tr>
</tbody>
</table>

#### Section 1.8 Properties of Real Numbers

**Commutative Properties**

Addition: \(a + b = b + a\) 
Multiplication: \(a \cdot b = b \cdot a\)

**Associative Properties**

Addition: \((a + b) + c = a + (b + c)\) 
Multiplication: \((a \cdot b) \cdot c = a \cdot (b \cdot c)\)

<table>
<thead>
<tr>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Add or multiply.</td>
</tr>
<tr>
<td>(3 + (-7) = -7 + 3)</td>
</tr>
<tr>
<td>(-8 \cdot 5 = 5 \cdot (-8))</td>
</tr>
<tr>
<td>((5 + 10) + 20 = 5 + (10 + 20))</td>
</tr>
<tr>
<td>((-3 \cdot 2) \cdot 11 = -3 \cdot (2 \cdot 11))</td>
</tr>
</tbody>
</table>

---

For use by Palm Beach State College only.
DEFINITIONS AND CONCEPTS

Section 1.8 Properties of Real Numbers (continued)

Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**. The reciprocal of a nonzero number \( a \) is \( \frac{1}{a} \) because \( a \cdot \frac{1}{a} = 1 \).

**Distributive Property**

\[ a(b + c) = a \cdot b + a \cdot c \]

**Identities**

\[ a + 0 = a \]
\[ 0 + a = a \]
\[ a \cdot 1 = a \]
\[ 1 \cdot a = a \]

**Inverses**

Additive or opposite: \( a + (-a) = 0 \)
Multiplicative or reciprocal: \( b \cdot \frac{1}{b} = 1 \)

The reciprocal of 3 is \( \frac{1}{3} \).
The reciprocal of \( -\frac{2}{5} \) is \( -\frac{5}{2} \).

\[ 8(6 + 10) = 5 \cdot 6 + 5 \cdot 10 \]
\[ -2(3 + x) = -2 \cdot 3 + (-2)(x) \]

\[ 5 + 0 = 5 \]
\[ 0 + (-2) = -2 \]
\[ -14 \cdot 1 = -14 \]
\[ 1 \cdot 27 = 27 \]

\[ 7 + (-7) = 0 \]
\[ \frac{3}{4} \cdot \frac{1}{3} = 1 \]

Chapter 1 Review

(1.2) **Insert <, >, or = in the appropriate space to make the following statements true.**

<table>
<thead>
<tr>
<th>1. 8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>2. 7</td>
<td>2</td>
</tr>
<tr>
<td>3. -4</td>
<td>-5</td>
</tr>
<tr>
<td>4. \frac{12}{2}</td>
<td>-8</td>
</tr>
<tr>
<td>5.</td>
<td>-7</td>
</tr>
<tr>
<td>6.</td>
<td>-9</td>
</tr>
<tr>
<td>7.</td>
<td>-1</td>
</tr>
<tr>
<td>8.</td>
<td>-14</td>
</tr>
<tr>
<td>9. 1.2</td>
<td>1.02</td>
</tr>
<tr>
<td>10. \frac{3}{2}</td>
<td>\frac{-3}{4}</td>
</tr>
</tbody>
</table>

**TRANSLATING**

**Translate each statement into symbols.**

11. Four is greater than or equal to negative three.
12. Six is not equal to five.
13. 0.03 is less than 0.3.
14. New York City has 155 museums and 400 art galleries. Write an inequality comparing the numbers 155 and 400. (Source: Absolute Trivia.com)

**Given the following sets of numbers, list the numbers in each set that also belong to the set of:**

- a. Natural numbers
- b. Whole numbers
- c. Integers
- d. Rational numbers
- e. Irrational numbers
- f. Real numbers

15. \{ -6, 0, 1, \frac{1}{2}, 3, \pi, 9.62 \}
16. \{ -3, -1.6, 2, 5, \frac{11}{2}, 15.1, \sqrt{5}, 2\pi \}

The following chart shows the gains and losses in dollars of Density Oil and Gas stock for a particular week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Gain or Loss in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>+1</td>
</tr>
<tr>
<td>Tuesday</td>
<td>-2</td>
</tr>
<tr>
<td>Wednesday</td>
<td>+5</td>
</tr>
<tr>
<td>Thursday</td>
<td>+1</td>
</tr>
<tr>
<td>Friday</td>
<td>-4</td>
</tr>
</tbody>
</table>

17. Which day showed the greatest loss?
18. Which day showed the greatest gain?

(1.3) **Write the number as a product of prime factors.**

19. 36
20. 120

**Perform the indicated operations. Write results in lowest terms.**

21. \frac{8}{15} \cdot \frac{27}{30}
22. \frac{7}{8} \div \frac{21}{32}
23. \frac{7}{15} + \frac{5}{6}
24. \frac{3}{4} - \frac{3}{20}
25. \frac{3}{4} + \frac{5}{8}
26. \frac{7}{5} - \frac{2}{3}
27. 5 \div \frac{1}{3}
28. 2 \cdot \frac{3}{4}

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Each circle represents a whole, or 1. Determine the unknown part of the circle.

29. \[ \frac{1}{6} \]
30. \[ \frac{3}{5} \]

Find the area and the perimeter of each figure.

\[ \triangle 31. \] 7/8 meter
\[ \triangle 32. \] \[ \frac{3}{10} \] in.

Octuplets were born in the U.S. in 2009. The following chart gives the octuplets’ birthweights. The babies are listed in order of birth.

<table>
<thead>
<tr>
<th>Baby</th>
<th>Gender</th>
<th>Birthweight (pounds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baby A</td>
<td>boy</td>
<td>2 \frac{7}{16}</td>
</tr>
<tr>
<td>Baby B</td>
<td>girl</td>
<td>1 \frac{1}{8}</td>
</tr>
<tr>
<td>Baby C</td>
<td>boy</td>
<td>3 \frac{11}{16}</td>
</tr>
<tr>
<td>Baby D</td>
<td>girl</td>
<td>2 \frac{3}{16}</td>
</tr>
<tr>
<td>Baby E</td>
<td>boy</td>
<td>1 \frac{3}{4}</td>
</tr>
<tr>
<td>Baby F</td>
<td>boy</td>
<td>2 \frac{9}{16}</td>
</tr>
<tr>
<td>Baby G</td>
<td>boy</td>
<td>1 \frac{13}{16}</td>
</tr>
<tr>
<td>Baby H</td>
<td>boy</td>
<td>2 \frac{7}{16}</td>
</tr>
</tbody>
</table>

33. What was the total weight of the boy octuplets?
34. What was the total weight of the girl octuplets?
35. Find the combined weight of all eight octuplets.
36. Which baby weighed the most?
37. Which baby weighed the least?
38. How much more did the heaviest baby weigh than the lightest baby?

**(1.4) Choose the correct answer for each statement.**

39. The expression \( 6 \cdot 3^2 + 2 \cdot 8 \) simplifies to
   a. \(-52 \)    b. \(448 \)   c. \(70 \)   d. \(64 \)
40. The expression \( 68 - 5 \cdot 2^3 \) simplifies to
   a. \(-232 \)   b. \(28 \)   c. \(38 \)   d. \(504 \)

**Simplify each expression.**

41. \( \left( \frac{2}{7} \right)^2 \)
42. \( \left( \frac{3}{4} \right)^3 \)
43. \( 3(1 + 2 \cdot 5) + 4 \)
44. \( 8 + 3(2 \cdot 6 - 1) \)
45. \( \frac{4 + |6 - 2| + 8^2}{4 + 6 \cdot 4} \)
46. \( 5(3(2 + 5) - 5) \)

**TRANSLATING**

Translate each word statement into symbols.

47. The difference of twenty and twelve is equal to the product of two and four.
48. The quotient of nine and two is greater than negative five.

Evaluate each expression if \( x = 6, y = 2, \) and \( z = 8. \)

49. \( 2x + 3y \)
50. \( x(y + 2z) \)
51. \( \frac{x}{y} + \frac{z}{2y} \)
52. \( x^2 - 3y^2 \)

**△ 53.** The expression \( 180 - a - b \) represents the measure of the unknown angle of the given triangle. Replace \( a \) with 37 and \( b \) with 80 to find the measure of the unknown angle.

**△ 54.** The expression \( 360 - a - b - c \) represents the measure of the unknown angle of the given quadrilateral. Replace \( a \) with 93, \( b \) with 80, and \( c \) with 82 to find the measure of the unknown angle.

**Decide whether the given number is a solution of the given equation.**

55. Is \( x = 3 \) a solution of \( 7x - 3 = 18? \)
56. Is \( x = 1 \) a solution of \( 3x^2 + 4 = x - 1? \)

**(1.5) Find the additive inverse or the opposite.**

57. \(-9\)  
58. \(\frac{2}{3}\)
59. \(\text{\text{\text{|-2|}}}\)
60. \(-\text{\text{|-7|}}\)

**Find the following sums.**

61. \(-15 + 4\)  
62. \(-6 + (-11)\)
63. \(\frac{1}{16} + \left(-\frac{1}{4}\right)\)
64. \(-8 + |{-3}|\)
65. \(-4.6 + (-9.3)\)
66. \(-2.8 + 6.7\)
(1.6) Perform the indicated operations.
67. \(6 - 20\) \hspace{1cm} 68. \(-3.1 - 8.4\)
69. \(-6 - (-11)\) \hspace{1cm} 70. \(4 - 15\)
71. \(-21 - 16 + 3(8 - 2)\) \hspace{1cm} 72. \(\frac{11 - (-9) + 6(8 - 2)}{2 + 3 \cdot 4}\)

Evaluate each expression for \(x = 3, y = -6,\) and \(z = -9.\) Then choose the correct evaluation.
73. \(2x^2 - y + z\)
   a. 15 \hspace{1cm} b. 3 \hspace{1cm} c. 27 \hspace{1cm} d. -3
74. \(\frac{|y - 4x|}{2x}\)
   a. 3 \hspace{1cm} b. 1 \hspace{1cm} c. -1 \hspace{1cm} d. -3

75. At the beginning of the week, the price of Density Oil and Gas stock from Exercises 17 and 18 is \$50 per share. Find the price of a share of stock at the end of the week.
76. Find the price of a share of stock by the end of the day on Wednesday.

(1.7) Find the multiplicative inverse or reciprocal.
77. \(-6\) \hspace{1cm} 78. \(\frac{3}{5}\)

Simplify each expression.
79. \(6(-8)\) \hspace{1cm} 80. \((-2)(-14)\)
81. \(-\frac{18}{-6}\) \hspace{1cm} 82. \(\frac{42}{-3}\)
83. \(\frac{4(-3) + (-8)}{2 + (-2)}\) \hspace{1cm} 84. \(\frac{3(-2)^2 - 5}{-14}\)
85. \(\frac{-6}{0}\) \hspace{1cm} 86. \(\frac{0}{-2}\)
87. \(-4^2 - (3 + 5) \div (-1) \cdot 2\)
88. \(-5^2 - (2 - 20) \div (-3) \cdot 3\)

If \(x = -5\) and \(y = -2,\) evaluate each expression.
89. \(x^2 - y^4\)
90. \(x^2 - y^3\)

(1.8) Name the property illustrated.
95. \(-6 + 5 = 5 + (-6)\)
96. \(6 \cdot 1 = 6\)
97. \(3(8 - 5) = 3 \cdot 8 - 3 \cdot (5)\)
98. \(4 + (-4) = 0\)
99. \(2 + (3 + 9) = (2 + 3) + 9\)
100. \(2 \cdot 8 = 8 \cdot 2\)
101. \(6(8 + 5) = 6 \cdot 8 + 6 \cdot 5\)
102. \((3 \cdot 8) \cdot 4 = 3 \cdot (8 \cdot 4)\)
103. \(4 \cdot \frac{1}{4} = 1\)
104. \(8 + 0 = 8\)

Use the distributive property to write each expression without parentheses.
105. \(5(y - 2)\)
106. \(-3(z + y)\)
107. \(-(7 - x + 4z)\)
108. \(\frac{1}{2}(6z - 10)\)
109. \(-4(3x + 5) - 7\)
110. \(-8(2y + 9) - 1\)

MIXED REVIEW

Insert <, >, or = in the space between each pair of numbers.
111. \([-11]\) \hspace{1cm} 112. \(\frac{1}{2}\) \hspace{1cm} \(\frac{7}{2}\)

Perform the indicated operations.
113. \(-7.2 + (-8.1)\)
114. \(14 - 20\)
115. \(4(-20)\)
116. \(-\frac{20}{4}\)
117. \(\frac{4}{3} \left(\frac{5}{16}\right)\)
118. \(-0.5(-0.3)\)
119. \(8 \div 2 \cdot 4\)
120. \((-2)^4\)
121. \(-\frac{3 - 2(-9)}{-15 - 3(-4)}\)
122. \(5 + 2[7 - 5]^2 + (1 - 3)]\)
123. \(\frac{-5}{8} + \frac{3}{4}\)
124. \(-\frac{15 + (-4)^2 + |-9|}{10 - 2 \cdot 5}\)

\(\triangle\) 125. A trim carpenter needs a piece of quarter round molding \(6\frac{1}{8}\) feet long for a bathroom. She finds a piece \(7\frac{1}{2}\) feet long.

How long a piece does she need to cut from the \(7\frac{1}{2}\)-foot-long molding in order to use it in the bathroom?
Chapter 1 Getting Ready for the Test

All the exercises below are Multiple Choice. Choose the correct letter(s). Also, letters may be used more than once.
Select the given operation between the two numbers.

1. For \(-5 + (-3)\), the operation is
   - A. addition
   - B. subtraction
   - C. multiplication
   - D. division

2. For \(-5(-3)\), the operation is
   - A. addition
   - B. subtraction
   - C. multiplication
   - D. division

Identify each as an
- A. equation
- B. expression

3. \(6x + 2 + 4x - 10\)
4. \(6x + 2 = 4x - 10\)

5. \(-2(x - 1) = 12\)
6. \(-7\left(x + \frac{1}{2}\right) = 22\)

For the exercises below, \(a\) and \(b\) are negative numbers. State whether each expression simplifies to
- A. positive number
- B. negative number
- C. 0
- D. not possible to determine

7. \(a + b\)
8. \(a \cdot b\)
9. \(\frac{a}{b}\)
10. \(a - 0\)
11. \(0 \cdot b\)
12. \(a - b\)
13. \(0 + b\)
14. \(\frac{0}{a}\)

The expression statement and the correct answer are given. Select the correct direction.
- A. Find the opposite.
- B. Find the reciprocal.
- C. Evaluate or simplify.

15. \(5\) Answer: \(\frac{1}{5}\)
16. \(3 + 2(-8)\) Answer: \(-13\)
17. \(2^3\) Answer: 8
18. \(-7\) Answer: 7

Translate the statement into symbols.

1. The absolute value of negative seven is greater than five.
2. The sum of nine and five is greater than or equal to four.

Simplify the expression.

3. \(-13 + 8\)
4. \(-13 - (-2)\)
5. \(12 \div 4 \cdot 3 - 6 \cdot 2\)
6. \((13)(-3)\)
7. \((-6)(-2)\)
8. \([-16]\)
9. \(-\frac{8}{0}\)
10. \([-6] + 2\)
11. \(\frac{1}{2} - \frac{5}{6}\)
12. \(\frac{3}{4} - \frac{1}{8}\)
13. \(-0.6 + 1.875\)
14. \(3(-4)^2 - 80\)
15. \(6[5 + 2(3 - 8) - 3]\)
16. \(-12 + 3 \cdot 8\)
17. \((-2)(0)(-3)\)

Insert \(<, >\), or \(=\) in the appropriate space to make each of the following statements true.

18. \(-3\) \(-7\)
19. \(4\) \(-8\)
20. \(2\) \(-3\)
21. \(-2\) \(-1\) \(-3\)
22. In the state of Massachusetts, there are 2221 licensed child care centers and 10,993 licensed home-based child care providers. Write an inequality statement comparing the numbers 2221 and 10,993. (Source: Children’s Foundation)
23. Given \(\{-5, -1, 0, \frac{1}{4}, 1, 7, 11.6, \sqrt{7}, 3\pi\}\), list the numbers in this set that also belong to the set of:
   - a. Natural numbers
   - b. Whole numbers
   - c. Integers
   - d. Rational numbers
   - e. Irrational numbers
   - f. Real numbers
   If \(x = 6, y = -2,\) and \(z = -3,\) evaluate each expression.
24. \(x^2 + y^2\)
25. \(x + yz\)
26. \(2 + 3x - y\)
27. \(\frac{y + z}{x}\)

Identify the property illustrated by each expression.
28. \(8 + (9 + 3) = (8 + 9) + 3\)
29. \(6 \cdot 8 = 8 \cdot 6\)
30. \(-6(2 + 4) = -6 \cdot 2 + (-6) \cdot 4\)

31. \(\frac{1}{6}(6) = 1\)

32. Find the opposite of \(-9\).

33. Find the reciprocal of \(-\frac{1}{3}\).

The New Orleans Saints were 22 yards from the goal when the following series of gains and losses occurred.

<table>
<thead>
<tr>
<th>Gains and Losses in Yards</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Down</td>
</tr>
<tr>
<td>Second Down</td>
</tr>
<tr>
<td>Third Down</td>
</tr>
<tr>
<td>Fourth Down</td>
</tr>
</tbody>
</table>

34. During which down did the greatest loss of yardage occur?

35. Was a touchdown scored?

36. The temperature at the Winter Olympics was a frigid 14 degrees below zero in the morning, but by noon it had risen 31 degrees. What was the temperature at noon?

37. A health insurance provider had net incomes of $356 million, $460 million, and $-166 million in 3 consecutive years. What was the health insurance provider’s total net income for these three years?

38. A stockbroker decided to sell 280 shares of stock, which decreased in value by $1.50 per share yesterday. How much money did she lose?