Can You Imagine a World Without the Internet?

In 1995, less than 1% of the world population was connected to the Internet. By 2015, that number had increased to 40%. Technology changes so fast that, if this trend continues, by the time you read this, far more than 40% of the world population will be connected to the Internet. The circle graph below shows Internet users by region of the world in 2015. In Section 5.2, Exercises 103 and 104, we explore more about the growth of Internet users.

Recall from Chapter 1 that an exponent is a shorthand notation for repeated factors. This chapter explores additional concepts about exponents and exponential expressions. An especially useful type of exponential expression is a polynomial. Polynomials model many real-world phenomena. In this chapter, we focus on polynomials and operations on polynomials.
5.1 Exponents

OBJECTIVES
1 Evaluate Exponential Expressions.
2 Use the Product Rule for Exponents.
3 Use the Power Rule for Exponents.
4 Use the Power Rules for Products and Quotients.
5 Use the Quotient Rule for Exponents, and Define a Number Raised to the 0 Power.
6 Decide Which Rule(s) to Use to Simplify an Expression.

OBJECTIVE 1 Evaluating Exponential Expressions

As we reviewed in Section 1.4, an exponent is a shorthand notation for repeated factors. For example, \( 2 \cdot 2 \cdot 2 \cdot 2 \) can be written as \( 2^4 \). The expression \( 2^5 \) is called an\( \text{exponential expression.} \) It is also called the fifth \( \text{power} \) of 2, or we say that 2 is \( \text{raised} \) to the fifth \( \text{power}. \)

\[ 5^6 = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \quad \text{and} \quad (-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) \]

6 factors; each factor is 5
4 factors; each factor is \(-3\)

The base of an exponential expression is the repeated factor. The \text{exponent} is the number of times that the base is used as a factor.

\[ 5^6 \quad \text{exponent} \]
\[ 5 \quad \text{base} \]
\[ (-3)^4 \quad \text{exponent} \]
\[ (-3) \quad \text{base} \]

EXAMPLE 1 Evaluate each expression.

\begin{align*}
\text{a. } 2^3 & \quad \text{b. } 3^1 \quad \text{c. } (-4)^2 \quad \text{d. } -4^2 \quad \text{e. } \left(\frac{1}{2}\right)^4 \quad \text{f. } (0.5)^3 \quad \text{g. } 4 \cdot 3^2
\end{align*}

Solution

\begin{align*}
\text{a. } 2^3 &= 2 \cdot 2 \cdot 2 = 8 \\
\text{b. } (-4)^2 &= (-4) \cdot (-4) = 16 \\
\text{c. } (\frac{1}{2})^4 &= \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16} \\
\text{d. } -4^2 &= -(4 \cdot 4) = -16 \\
\text{e. } (0.5)^3 &= (0.5) \cdot (0.5) \cdot (0.5) = 0.125 \\
\text{f. } 4 \cdot 3^2 &= 4 \cdot 9 = 36
\end{align*}

PRACTICE 1 Evaluate each expression.

\begin{align*}
\text{a. } 3^3 & \quad \text{b. } 4^1 \quad \text{c. } (-8)^2 \quad \text{d. } -8^2 \\
\text{e. } \left(\frac{3}{4}\right)^3 & \quad \text{f. } (0.3)^4 \quad \text{g. } 3 \cdot 5^2
\end{align*}

Notice how similar \(-4^2\) is to \((-4)^2\) in the example above. The difference between the two is the parentheses. In \((-4)^2\), the parentheses tell us that the base, or repeated factor, is \(-4\). In \(-4^2\), only 4 is the base.

Helpful Hint

Be careful when identifying the base of an exponential expression. Pay close attention to the use of parentheses.

\[
\begin{array}{ccc}
(-3)^2 & -3^2 & 2 \cdot 3^2 \\
\text{The base is } -3. & \text{The base is } 3. & \text{The base is } 3.
\end{array}
\]

\[
\begin{array}{ccc}
(-3)^2 = (-3) \cdot (-3) & -3^2 = -(3 \cdot 3) & 2 \cdot 3^2 = 2 \cdot 3 \cdot 3 = 18
\end{array}
\]

An exponent has the same meaning whether the base is a number or a variable. If \(x\) is a real number and \(n\) is a positive integer, then \(x^n\) is the product of \(n\) factors, each of which is \(x\).

\[x^n = x \cdot x \cdot x \cdot x \cdot \ldots \cdot x\]

\(n\) factors of \(x\)
EXAMPLE 2 Evaluate each expression for the given value of \( x \).

a. \( 2^3; \ x \) is 5

\[ 2^3 = 2 \cdot (5)^3 = 2 \cdot (5 \cdot 5 \cdot 5) = 2 \cdot 125 = 250 \]

b. \( \frac{9}{x^7}; \ x \) is \(-3\)

\[ \frac{9}{x^7} = \frac{9}{(-3)^7} = \frac{9}{(-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3) \cdot (-3)} = \frac{9}{9} = 1 \]

PRACTICE Evaluate each expression for the given value of \( x \).

a. \( 3^4; \ x \) is 3

b. \( \frac{6}{x^5}; \ x \) is \(-4\)

OBJECTIVE Using the Product Rule

Exponential expressions can be multiplied, divided, added, subtracted, and themselves raised to powers. By our definition of an exponent,

\[ 5^4 \cdot 5^3 = \underbrace{5 \cdot 5 \cdot 5 \cdot 5} \cdot \underbrace{5 \cdot 5 \cdot 5} = 5^7 \]

Also,

\[ x^2 \cdot x^3 = (x \cdot x) \cdot (x \cdot x \cdot x) = x^5 \]

In both cases, notice that the result is exactly the same if the exponents are added.

\[ 5^4 \cdot 5^3 = 5^{4+3} = 5^7 \quad \text{and} \quad x^2 \cdot x^3 = x^{2+3} = x^5 \]

This suggests the following rule.

**Product Rule for Exponents**

If \( m \) and \( n \) are positive integers and \( a \) is a real number, then

\[ a^m \cdot a^n = a^{m+n} \quad \text{Keep common base.} \]

For example, \( 3^5 \cdot 3^7 = 3^{5+7} = 3^{12} \quad \text{Keep common base.} \)

**Helpful Hint**

Don't forget that

\[ 3^5 \cdot 3^7 \neq 9^{12} \quad \text{Common base not kept.} \]

\[ 3^5 \cdot 3^7 = \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} \cdot \underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = 3^{12} \quad \text{12 factors of 3, not 9} \]
In other words, to multiply two exponential expressions with the same base, we keep the base and add the exponents. We call this simplifying the exponential expression.

**Example 3** Use the product rule to simplify.

a. \(4^2 \cdot 4^3\)  
b. \(x^4 \cdot x^6\)  
c. \(y^3 \cdot y\)  
d. \(y^3 \cdot y^2 \cdot y^7\)  
e. \((-5)^7 \cdot (-5)^8\)  
f. \(a^2 \cdot b^2\)

**Solution**

a. \(4^{2+3} = 4^5\)  
   Keep common base.

b. \(x^{4+6} = x^{10}\)

d. \(y^{3+2+7} = y^{12}\)

e. \((-5)^{7+8} = (-5)^{15}\)

f. \(a^2 \cdot b^2\) Cannot be simplified because \(a\) and \(b\) are different bases.

**Practice 3** Use the product rule to simplify.

a. \(3^4 \cdot 3^6\)  
b. \(y^3 \cdot y^2\)  
c. \(z \cdot z^4\)  
d. \(x^3 \cdot x^2 \cdot x^6\)  
e. \((-2)^5 \cdot (-2)^3\)  
f. \(b^3 \cdot b^5\)

**Concept Check**

Where possible, use the product rule to simplify the expression.

a. \(z^2 \cdot z^{14}\)  
b. \(x^2 \cdot y^{14}\)  
c. \(9^8 \cdot 9^5\)  
d. \(9^8 \cdot 2^7\)

**Example 4** Use the product rule to simplify \((2x^2 \cdot -3x^5)\).

**Solution**

Recall that \(2x^2\) means \(2 \cdot x^2\) and \(-3x^5\) means \(-3 \cdot x^5\).

\[
(2x^2)(-3x^5) = 2 \cdot x^2 \cdot -3 \cdot x^5
\]

Remove parentheses.

\[
= 2 \cdot -3 \cdot x^2 \cdot x^5
\]

Group factors with common bases.

\[
= -6x^7
\]

Simplify.

**Practice 4** Use the product rule to simplify \((-5y^3) \cdot (-3y^4)\).

**Example 5** Simplify.

a. \((x^2 y)(x^3 y^2)\)  
b. \((-a^7 b^4)(3ab^9)\)

**Solution**

a. \((x^2 y)(x^3 y^2) = (x^2 \cdot x^3) \cdot (y^1 \cdot y^2)\)  
   Group like bases and write \(y\) as \(y^1\).

\[
= x^5 \cdot y^3
\]

b. \((-a^7 b^4)(3ab^9) = (-1 \cdot 3) \cdot (a^7 \cdot a^1) \cdot (b^1 \cdot b^9)\)

\[
= -3a^{13}b^{13}
\]

**Practice 5** Simplify.

a. \((y^7 z^3)(y^5 z)\)  
b. \((-m^4 n^4)(7mn^{10})\)

Answers to Concept Check:

a. \(z^{16}\)  
b. cannot be simplified  
c. \(9^{11}\)  
d. cannot be simplified
Helpful Hint

These examples will remind you of the difference between adding and multiplying terms.

Addition

\[ 5x^3 + 3x^3 = (5 + 3)x^3 = 8x^3 \quad \text{By the distributive property.} \]
\[ 7x + 4x^2 = 7x + 4x^2 \quad \text{Cannot be combined.} \]

Multiplication

\[ (5x)(3x^3) = 5 \cdot 3 \cdot x \cdot x^3 = 15x^4 \quad \text{By the product rule.} \]
\[ (7x)(4x^2) = 7 \cdot 4 \cdot x \cdot x^2 = 28x^3 \quad \text{By the product rule.} \]

Objective 3

Using the Power Rule

Exponential expressions can themselves be raised to powers. Let’s try to discover a rule that simplifies an expression like \((x^2)^3\). By definition,

\[ (x^2)^3 = (x^2)(x^2)(x^2) \]

which can be simplified by the product rule for exponents.

\[ (x^2)^3 = (x^2)(x^2)(x^2) = x^{2+2+2} = x^6 \]

Notice that the result is exactly the same if we multiply the exponents.

\[ (x^2)^3 = x^{2 \cdot 3} = x^6 \]

The following property states this result.

Power Rule for Exponents

If \(m\) and \(n\) are positive integers and \(a\) is a real number, then

\[ (a^m)^n = a^{mn} \quad \text{Multiply exponents.} \]

Keep common base.

For example,

\[ (7^2)^5 = 7^{2 \cdot 5} = 7^{10} \quad \text{Multiply exponents.} \]

Keep common base.

To raise a power to a power, keep the base and multiply the exponents.

Example 6

Use the power rule to simplify.

a. \((y^8)^2\)    b. \((8^4)^5\)    c. \((-5)^3\)^7

Solution

a. \((y^8)^2 = y^{8 \cdot 2} = y^{16}\)    b. \((8^4)^5 = 8^{4 \cdot 5} = 8^{20}\)    c. \((-5)^3\)^7 = \((-5)^{21}\)

Practice 6

Use the power rule to simplify.

a. \((z^3)^7\)    b. \((4^9)^2\)    c. \((-2)^3\)^5

Helpful Hint

Take a moment to make sure that you understand when to apply the product rule and when to apply the power rule.

<table>
<thead>
<tr>
<th>Product Rule → Add Exponents</th>
<th>Power Rule → Multiply Exponents</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^5 \cdot x^7 = x^{5+7} = x^{12})</td>
<td>((x^5)^7 = x^{5 \cdot 7} = x^{35})</td>
</tr>
<tr>
<td>(y^6 \cdot y^2 = y^{6+2} = y^8)</td>
<td>((y^6)^2 = y^{6 \cdot 2} = y^{12})</td>
</tr>
</tbody>
</table>

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For example, \( (xy)^3 \), for example,
\[
(xy)^3 = (xy)(xy)(xy) = x \cdot x \cdot x \cdot y \cdot y \cdot y = x^3y^3
\]

Notice that to simplify the expression \( (xy)^3 \), we raise each factor within the parentheses to a power of 3.

\[
(xy)^3 = x^3y^3
\]

In general, we have the following rule.

**Power of a Product Rule**

If \( n \) is a positive integer and \( a \) and \( b \) are real numbers, then

\[
(ab)^n = a^n b^n
\]

For example, \((3x)^5 = 3^5x^5\).

In other words, to raise a product to a power, we raise each factor to the power.

**Example 7**

Simplify each expression.

- \( (st)^4 \)
- \( (2a)^3 \)
- \( \left( \frac{1}{3}mn^3 \right)^2 \)
- \( (-5x^3y^3z)^2 \)

**Solution**

- \( (st)^4 = s^4 \cdot t^4 = s^4t^4 \) Use the power of a product rule.
- \( (2a)^3 = 2^3 \cdot a^3 = 8a^3 \) Use the power of a product rule.
- \( \left( \frac{1}{3}mn^3 \right)^2 = \left( \frac{1}{3} \right)^2 \cdot (m)^2 \cdot (n^3)^2 = \frac{1}{9}m^2n^6 \) Use the power of a product rule.
- \( (-5x^3y^3z)^2 = (-5)^2 \cdot (x^2)^2 \cdot (y^3)^2 \cdot (z^1)^2 = 25x^6y^6z^2 \) Use the power rule for exponents.

**Practice 7**

Simplify each expression.

- \( (pr)^5 \)
- \( (6b)^2 \)
- \( \left( \frac{1}{4}x^2y \right)^3 \)
- \( (-3a^3b^4c)^4 \)

Let’s see what happens when we raise a quotient to a power. To simplify \( \left( \frac{x}{y} \right)^3 \), for example,
\[
\left( \frac{x}{y} \right)^3 = \left( \frac{x}{y} \right) \left( \frac{x}{y} \right) \left( \frac{x}{y} \right) = \frac{x \cdot x \cdot x}{y \cdot y \cdot y} \text{ Multiply fractions.}
\]
\[
= \frac{x^3}{y^3} \text{ Simplify.}
\]

Notice that to simplify the expression \( \left( \frac{x}{y} \right)^3 \), we raise both the numerator and the denominator to a power of 3.

\[
\left( \frac{x}{y} \right)^3 = \frac{x^3}{y^3}
\]
In general, we have the following.

**Power of a Quotient Rule**

If \( n \) is a positive integer and \( a \) and \( c \) are real numbers, then

\[
\left( \frac{a}{c} \right)^n = \frac{a^n}{c^n}, \quad c \neq 0
\]

For example, \( \left( \frac{y}{7} \right)^4 = \frac{y^4}{7^4} \).

In other words, to raise a quotient to a power, we raise both the numerator and the denominator to the power.

**EXAMPLE 8** Simplify each expression.

\begin{align*}
\text{a. } & \left( \frac{m}{n} \right)^7 \\
\text{b. } & \left( \frac{x^3}{3y^5} \right)^4
\end{align*}

**Solution**

\begin{align*}
\text{a. } & \left( \frac{m}{n} \right)^7 = \frac{m^7}{n^7}, \quad n \neq 0 \quad \text{Use the power of a quotient rule.} \\
\text{b. } & \left( \frac{x^3}{3y^5} \right)^4 = \frac{(x^3)^4}{3^4 \cdot (y^5)^4}, \quad y \neq 0 \quad \text{Use the power of a product or quotient rule.} \\
& = \frac{x^{12}}{81y^{20}} \quad \text{Use the power rule for exponents.}
\end{align*}

**PRACTICE 8** Simplify each expression.

\begin{align*}
\text{a. } & \left( \frac{x^5}{y^2} \right)^5 \\
\text{b. } & \left( \frac{2a^4}{b^3} \right)^5
\end{align*}

**OBJECTIVE 5 Using the Quotient Rule and Defining the Zero Exponent**

Another pattern for simplifying exponential expressions involves quotients.

To simplify an expression like \( \frac{x^5}{x^3} \), in which the numerator and the denominator have a common base, we can apply the fundamental principle of fractions and divide the numerator and the denominator by the common base factors. Assume for the remainder of this section that denominators are not 0.

\[
\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x}
= x^2
\]

Notice that the result is exactly the same if we subtract exponents of the common bases.

\[
\frac{x^5}{x^3} = x^{5-3} = x^2
\]

The quotient rule for exponents states this result in a general way.
For example, \( \frac{x^6}{x^2} = x^{6-2} = x^4 \).

In other words, to divide one exponential expression by another with a common base, keep the base and subtract exponents.

**Quotient Rule for Exponents**

If \( m \) and \( n \) are positive integers and \( a \) is a real number, then

\[
\frac{a^m}{a^n} = a^{m-n}
\]

as long as \( a \) is not 0.

**Example 9** Simplify each quotient.

- a. \( \frac{x^5}{x^2} \)
- b. \( \frac{4^7}{4^3} \)
- c. \( \frac{(-3)^5}{(-3)^2} \)
- d. \( \frac{s^2}{r^3} \)
- e. \( \frac{2^x y^2}{xy} \)

**Solution**

- a. \( \frac{x^5}{x^2} = x^{5-2} = x^3 \) Use the quotient rule.
- b. \( \frac{4^7}{4^3} = 4^{7-3} = 4^4 = 256 \) Use the quotient rule.
- c. \( \frac{(-3)^5}{(-3)^2} = (-3)^{5-2} = -27 \) Use the quotient rule.
- d. \( \frac{s^2}{r^3} \) Cannot be simplified because \( s \) and \( r \) are different bases.
- e. Begin by grouping common bases.
  \[
  \frac{2x^3y^2}{xy} = 2 \cdot \frac{x^3 \cdot y^2}{x \cdot y} = 2 \cdot (x^{3-1}) \cdot (y^{2-1}) 
  \]
  \[
  = 2x^2y^1 \text{ or } 2x^2y
  \]
  Use the quotient rule.

**Practice 9** Simplify each quotient.

- a. \( \frac{z^8}{z^4} \)
- b. \( \frac{(-5)^5}{(-5)^3} \)
- c. \( \frac{8^8}{8^6} \)
- d. \( \frac{q^5}{r^2} \)
- e. \( \frac{6x^3y^7}{xy^5} \)

**Concept Check**

Suppose you are simplifying each expression. Tell whether you would add the exponents, subtract the exponents, multiply the exponents, divide the exponents, or none of these.

- a. \( (x^3)^{21} \)
- b. \( \frac{y^{15}}{y^3} \)
- c. \( z^{16} + z^8 \)
- d. \( w^{45} \cdot w^9 \)

Let’s now give meaning to an expression such as \( x^0 \). To do so, we will simplify \( \frac{x^3}{x^3} \) in two ways and compare the results.

\[
\frac{x^3}{x^3} = x^{3-3} = x^0 \quad \text{Apply the quotient rule.}
\]

\[
\frac{x^3}{x^3} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x} = 1 \quad \text{Apply the fundamental principle for fractions.}
\]
Since $\frac{x^3}{x^3} = x^0$ and $\frac{x^3}{x^3} = 1$, we define that $x^0 = 1$ as long as $x$ is not 0.

**Zero Exponent**
$a^0 = 1$, as long as $a$ is not 0.

In other words, any base raised to the 0 power is 1 as long as the base is not 0.

**Example 10** Simplify each expression.

- a. $3^0$
- b. $(5x^3y^2)^0$
- c. $(-5)^0$
- d. $-5^0$
- e. $\left(\frac{3}{100}\right)^0$
- f. $4x^0$

**Solution**

- a. $3^0 = 1$
- b. Assume that neither $x$ nor $y$ is zero.
  
  $(5x^3y^2)^0 = 1$
- c. $(-5)^0 = 1$
- d. $-5^0 = -1 \cdot 5^0 = -1 \cdot 1 = -1$
- e. $\left(\frac{3}{100}\right)^0 = 1$
- f. $4x^0 = 4 \cdot x^0 = 4 \cdot 1 = 4$

**Practice 10** Simplify the following expressions.

- a. $-3^0$
- b. $(-3)^0$
- c. $8^0$
- d. $(0.2)^0$
- e. $(7a^2y^4)^0$
- f. $7y^0$

**Objective 6 Deciding Which Rule to Use**

Let’s practice deciding which rule(s) to use to simplify. We will continue this discussion with more examples in Section 5.5.

**Example 11** Simplify each expression.

- a. $x^7 \cdot x^4$
- b. $\left(\frac{t}{2}\right)^4$
- c. $(9y^5)^2$

**Solution**

- a. Here we have a product, so we use the product rule to simplify.
  
  $x^7 \cdot x^4 = x^{7+4} = x^{11}$
- b. This is a quotient raised to a power, so we use the power of a quotient rule.
  
  $\left(\frac{t}{2}\right)^4 = \frac{t^4}{2^4} = \frac{t^4}{16}$
- c. This is a product raised to a power, so we use the power of a product rule.
  
  $(9y^5)^2 = 9^2(y^5)^2 = 81y^{10}$

**Practice 11**

- a. $\left(\frac{x}{12}\right)^2$
- b. $(4x^6)^3$
- c. $y^{10} \cdot y^3$
EXAMPLE 12 Simplify each expression.

a. $4^2 - 4^0$

b. $(x^0)^3 + (2^0)^5$

c. $\left(\frac{3y^7}{6x^5}\right)^2$

d. $(2a^3b^4)^3 - 8a^6b^2$

**Solution**

a. $4^2 - 4^0 = 16 - 1 = 15$

Remember that $4^0 = 1$.

b. $(x^0)^3 + (2^0)^5 = 1^3 + 1^5 = 1 + 1 = 2$

c. $\left(\frac{3y^7}{6x^5}\right)^2 = \frac{3^2(y^7)^2}{6^2(x^5)^2} = 9 \cdot y^{14} = \frac{y^{14}}{36 \cdot x^{10}} = \frac{y^{14}}{4x^{10}}$

d. $(2a^3b^4)^3 - 8a^6b^2 = 2^3(a^3)^3(b^4)^3 = \frac{8a^{12}b^{12}}{-8a^6b^2} = -1 \cdot (a^{9-9}) \cdot (b^{12-2})$

$$= -1 \cdot a^0 \cdot b^{10} = -1 \cdot 1 \cdot b^{10} = -b^{10}$$

**Practice**

12. Simplify each expression.

a. $8^2 - 8^0$

b. $(x^0)^6 + (4^0)^5$

c. $\left(\frac{5x^3}{15y^4}\right)^2$

d. $(2z^8x^5)^4 - 16z^2x^{20}$

**Vocabulary, Readiness & Video Check**

Use the choices below to fill in each blank. Some choices may be used more than once.

0 base add
1 exponent multiply

1. Repeated multiplication of the same factor can be written using a(n) ________.
2. In $5^2$, the 2 is called the ________ and the 5 is called the ________.
3. To simplify $x^2 \cdot x^7$, keep the base and ________ the exponents.
4. To simplify $(x^3)^6$, keep the base and ________ the exponents.
5. The understood exponent on the term $y$ is ________.
6. If $x^0 = 1$, the exponent is ________.

**Martin-Gay Interactive Videos**

Watch the section lecture video and answer the following questions.

7. Examples 3 and 4 illustrate how to find the base of an exponential expression both with and without parentheses. Explain how identifying the base of Example 7 is similar to identifying the base of Example 4.

8. Why were the commutative and associative properties applied in Example 12? Were these properties used in another example?

9. What point is made at the end of Example 15?

10. Although it’s not especially emphasized in Example 20, what is helpful to remind yourself about the $-2$ in the problem?

11. In Example 24, which exponent rule is used to show that any non-zero base raised to zero is 1?

12. When simplifying an exponential expression that’s a fraction, will you always use the quotient rule? Refer to Example 30 for this objective to support your answer.
For each of the following expressions, state the exponent shown and its corresponding base.

1. $3^2$
2. $(-3)^6$
3. $-4^2$
4. $5 \cdot 3^4$
5. $5x^2$
6. $(5x)^3$

Evaluate each expression. See Example 1.

7. $7^2$
8. $-3^2$
9. $(-5)^1$
10. $(-3)^2$
11. $-2^4$
12. $-4^3$
13. $(-2)^4$
14. $(-4)^3$
15. $(0.1)^5$
16. $(0.2)^5$
17. $\left(\frac{1}{7}\right)^4$
18. $\left(-\frac{1}{9}\right)^2$
19. $7 \cdot 2^5$
20. $9 \cdot 1^7$
21. $-2 \cdot 5^3$
22. $-4 \cdot 3^3$

Evaluate each expression for the replacement values given. See Example 2.

23. $x^2; x = -2$
24. $x^3; x = -2$
25. $5x^3; x = 3$
26. $4x^2; x = -1$
27. $2x y^2; x = 3$ and $y = 5$
28. $-4x y^3; x = 2$ and $y = -1$
29. $\frac{2x^4}{5}; x = -2$
30. $\frac{10}{3y^3}; y = 5$

Use the product rule to simplify each expression. Write the results using exponents. See Examples 3 through 5.

31. $x^2 \cdot x^5$
32. $y^3 \cdot y$
33. $(-3)^3 \cdot (-3)^9$
34. $(-5)^7 \cdot (-5)^6$
36. $(-2x^4) (-2z^2)$
38. $(a^b)(a^{13b})$
39. $(-8mn^6)(9m^7n^2)$
40. $(-7a^3b^3)(7a^5b)$
41. $(4x^{10}) (-6z^7)(z^3)$
42. $(12x^5) (-x^6)(x^4)$

43. The rectangle below has width $4x^2$ feet and length $5x^3$ feet. Find its area as an expression in $x$. $(A = l \cdot w)$

44. The parallelogram below has base length $9y^2$ meters and height $2y^{10}$ meters. Find its area as an expression in $y$. $(A = b \cdot h)$

MIXED PRACTICE

Use the power rule and the power of a product or quotient rule to simplify each expression. See Examples 6 through 8.

45. $(x^2)^4$
46. $(y^3)^5$
47. $(pq)^8$
48. $(ab)^6$
49. $(2x^3)^3$
50. $(4x^6)^2$
51. $(x^2y^3)^5$
52. $(a^5b)^7$
53. $(-7a^2b^5c)^2$
54. $(-3x^3y^2z^3)^3$
55. $\left(\frac{r}{s}\right)^9$
56. $\left(\frac{q}{t}\right)^{11}$
57. $(\frac{mp}{n})^5$
58. $(\frac{x^2}{y^2})^2$
59. $\left(-\frac{2x^2}{y^3}\right)^2$
60. $\left(\frac{xy^4}{x^3}ight)^3$

61. The square shown has sides of length $8z^5$ decimeters. Find its area. $(A = s^2)$

62. Given the circle below with radius $5y$ centimeters, find its area. Do not approximate $\pi$. $(A = \pi r^2)$

63. The vault below is in the shape of a cube. If each side is $3y^4$ feet, find its volume. $(V = s^3)$

64. The silo shown is in the shape of a cylinder. If its radius is $4x$ meters and its height is $5x^3$ meters, find its volume. Do not approximate $\pi$. $(V = \pi r^2h)$
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Section 5.1 Exponents

Use the quotient rule and simplify each expression. See Example 9.

65. \( \frac{x^8}{x} \)  

66. \( \frac{y^{10}}{y^9} \)

67. \( \frac{(-4)^6}{(-4)^3} \)

68. \( \frac{(-6)^{13}}{(-6)^{11}} \)

69. \( \frac{p^7q^{20}}{pq^{13}} \)

70. \( \frac{x^8y^6}{xy^5} \)

71. \( \frac{7x^5y^5}{14x^7y^2} \)

72. \( \frac{9a^2b^7}{27ab^2} \)

Simplify each expression. See Example 10.

73. \( 7^0 \)  

74. \( 23^0 \)  

75. \( (2x)^0 \)

76. \( (4y)^0 \)  

77. \( -7x^0 \)  

78. \( -2x^0 \)

79. \( 5^0 + y^0 \)  

80. \( -3^0 + 4^0 \)

MIXED PRACTICE

Simplify each expression. See Examples 1 through 12.

81. \(-9^2\)  

82. \((9)^2\)

83. \(\left(\frac{1}{4}\right)^3\)  

84. \(\left(\frac{2}{3}\right)^3\)

85. \(b^3b^2\)  

86. \(y^y\)

87. \(a^2a^3a^4\)

88. \(x^2x^4x^9\)

89. \(2x^2)(-8x^4)\)  

90. \((3y^3)(-5y)\)

91. \((a^2b^2)(a^2b^3)\)

92. \((y^2z^3)(y^1z^13)\)

93. \((-2mn^6)(-13m^3n)\)  

94. \((-3z^2)(-7st^{10})\)

95. \((x^4)^{10}\)  

96. \((x^5)^{11}\)

97. \((4ab)^3\)  

98. \((2ab)^4\)

99. \((-6x^2y)^2\)  

100. \((-3xy^2a)^3\)

101. \(\frac{3x^3}{x^4}\)

102. \(\frac{5a^9}{x^3}\)

103. \((9xy)^2\)  

104. \((2ab)^5\)

105. \(2^3 + 2^0\)  

106. \(7^2 - 7^0\)

107. \(\left(\frac{3y^5}{6x}\right)^3\)  

108. \(\frac{2ab}{6yz}\)

109. \(2x^3y^2z\)  

110. \(\frac{x^{12}y^{13}}{x^3y^5}\)

111. \((5^0)^3 + (y^0)^7\)  

112. \((9^0)^4 + (x^0)^5\)

113. \(\left(\frac{5x^9}{10y^{11}}\right)^2\)  

114. \(\frac{3x^4y^2}{9z^7}\)

115. \((2a^2b)^4\)  

116. \(\frac{(2x^5y^2z)^5}{-32x^{20}y^{10}}\)

REVIEW AND PREVIEW

Simplify each expression by combining any like terms. Use the distributive property to remove any parentheses. See Section 2.1.

117. \(y - 10 + y\)

118. \(-6z + 20 - 3z\)

119. \(7x + 2 - 8x - 6\)

120. \(10y - 14 - y - 14\)

121. \(2(x - 5) + 3(5 - x)\)

122. \(-3(w + 7) + 5(w + 1)\)

CONCEPT EXTENSIONS

Solve. See the Concept Checks in this section. For Exercises 123 through 126, match the expression with the operation needed to simplify each. A letter may be used more than once and a letter may not be used at all.

123. \((x^4)^{23}\)  

124. \(x^{14} \cdot x^{23}\)

125. \(x^{14} + x^{23}\)

126. \(\frac{x^{35}}{x^{17}}\)

A. Add the exponents  

B. Subtract the exponents  

C. Multiply the exponents  

D. Divide the exponents  

E. None of these

Fill in the boxes so that each statement is true. (More than one answer is possible for each exercise.)

127. \(a^9 \cdot a^9 = a^{12}\)  

128. \(a^4 \cdot a^3 = a^{11}\)

129. \(\frac{a^5}{a^3} = a^2\)

130. \(a^3 \cdot a^4 = a^{10}\)

\(\Delta\) 131. The formula \(V = x^3\) can be used to find the volume \(V\) of a cube with side length \(x\). Find the volume of a cube with side length 7 meters. (Volume is measured in cubic units.)

\(\Delta\) 132. The formula \(S = 6x^2\) can be used to find the surface area \(S\) of a cube with side length \(x\). Find the surface area of a cube with side length 5 meters. (Surface area is measured in square units.)

\(\Delta\) 133. To find the amount of water that a swimming pool in the shape of a cube can hold, do we use the formula for volume of the cube or surface area of the cube? (See Exercises 131 and 132.)

\(\Delta\) 134. To find the amount of material needed to cover an ottoman in the shape of a cube, do we use the formula for volume of the cube or surface area of the cube? (See Exercises 131 and 132.)

\(\Delta\) 135. Explain why \((-5)^{3} = 625\), while \((-5)^{7} = -625\).

\(\Delta\) 136. Explain why \(5 \cdot 4^2 = 80\), while \((5 \cdot 4)^2 = 400\).

\(\Delta\) 137. In your own words, explain why \(s^0 = 1\).

\(\Delta\) 138. In your own words, explain when \((-3)^n\) is positive and when it is negative.

Simplify each expression. Assume that variables represent positive integers.

139. \(x^{5a} \cdot x^{4a}\)  

140. \(b^{30}b^{4a}\)

141. \((ab)^5\)

142. \((2a^3b^2)^4\)  

143. \(x^8\)

144. \(x^{15b}\)

Solve. Round money amounts to 2 decimal places.

145. Suppose you borrow money for 6 months. If the interest is compounded monthly, the formula \(A = P\left(1 + \frac{r}{12}\right)^6\) gives the total amount \(A\) to be repaid at the end of 6 months. For a loan of \(P = 1000\) and interest rate of 9\% \((r = 0.09)\), how much money is needed to pay off the loan?

146. Suppose you borrow money for 3 years. If the interest is compounded quarterly, the formula \(A = P\left(1 + \frac{r}{4}\right)^{12}\) gives the total amount \(A\) to be repaid at the end of 3 years. For a loan of \$10,000 and interest rate of 8\% \((r = 0.08)\), how much money is needed to pay off the loan in 3 years?

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**5.2 Adding and Subtracting Polynomials**

**OBJECTIVES**

1. Define Polynomial, Monomial, Binomial, Trinomial, and Degree.
2. Find the Value of a Polynomial Given Replacement Values for the Variables.
3. Simplify a Polynomial by Combining Like Terms.
4. Add and Subtract Polynomials.

**OBJECTIVE 1**

**Defining Polynomial, Monomial, Binomial, Trinomial, and Degree**

In this section, we introduce a special algebraic expression called a polynomial. Let’s first review some definitions presented in Section 2.1.

Recall that a *term* is a number or the product of a number and variables raised to powers. The terms of the expression \(4x^2 + 3x\) are \(4x^2\) and \(3x\).

The terms of the expression \(9x^4 - 7x - 1\) are \(9x^4\), \(-7x\), and \(-1\).

The **numerical coefficient** of a term, or simply the **coefficient**, is the numerical factor of each term. If no numerical factor appears in the term, then the coefficient is understood to be 1. If the term is a number only, it is called a **constant term** or simply a constant.

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^5)</td>
<td>1</td>
</tr>
<tr>
<td>(3x^2)</td>
<td>3</td>
</tr>
<tr>
<td>(-4x)</td>
<td>-4</td>
</tr>
<tr>
<td>(-x^2y)</td>
<td>-1</td>
</tr>
<tr>
<td>3 (constant)</td>
<td>3</td>
</tr>
</tbody>
</table>

Now we are ready to define a polynomial.

**Polynomial**

A *polynomial in* \(x\) is a finite sum of terms of the form \(ax^n\), where \(a\) is a real number and \(n\) is a whole number.

For example,

\[x^5 - 3x^3 + 2x^2 - 5x + 1\]

is a polynomial. Notice that this polynomial is written in **descending powers** of \(x\) because the powers of \(x\) decrease from left to right. (Recall that the term 1 can be thought of as \(1x^0\).)

On the other hand,

\[x^{-5} + 2x - 3\]

is not a polynomial because it contains an exponent, \(-5\), that is not a whole number.

(We study negative exponents in Section 5.5 of this chapter.)

Some polynomials are given special names.

**Types of Polynomials**

- A **monomial** is a polynomial with exactly one term.
- A **binomial** is a polynomial with exactly two terms.
- A **trinomial** is a polynomial with exactly three terms.
The following are examples of monomials, binomials, and trinomials. Each of these examples is also a polynomial.

### POLYNOMIALS

<table>
<thead>
<tr>
<th>Monomials</th>
<th>Binomials</th>
<th>Trinomials</th>
<th>More than Three Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ax^2 )</td>
<td>( x + y )</td>
<td>( x^2 + 4xy + y^2 )</td>
<td>( 5x^3 - 6x^2 + 3x - 6 )</td>
</tr>
<tr>
<td>(-3z)</td>
<td>( 3p + 2 )</td>
<td>( x^5 + 7x^2 - x )</td>
<td>( -y^5 + y^4 - 3y^3 - y^2 + y )</td>
</tr>
<tr>
<td>( 4 )</td>
<td>( 4x^2 - 7 )</td>
<td>( -q^4 + q^3 - 2q )</td>
<td>( x^6 + x^4 - x^3 + 1 )</td>
</tr>
</tbody>
</table>

Each term of a polynomial has a **degree**.

**Degree of a Term**

The degree of a term is the sum of the exponents on the variables contained in the term.

**EXAMPLE 1** Find the degree of each term.

a. \(-3x^2\)  
b. \(5x^3yz\)  
c. 2

**Solution**

a. The exponent on \(x\) is 2, so the degree of the term is 2.

b. \(5x^3yz\) can be written as \(5x^1y^1z^1\). The degree of the term is the sum of the exponents on its variables, so the degree is \(3 + 1 + 1\) or 5.

c. The constant, 2, can be written as \(2x^0\) (since \(x^0 = 1\)). The degree of 2 or \(2x^0\) is 0.

**PRACTICE**

Find the degree of each term.

a. \(5y^3\)  
b. \(-3a^2b^5c\)  
c. 8

From the preceding, we can say that **the degree of a constant is 0**.

Each polynomial also has a degree.

**Degree of a Polynomial**

The degree of a polynomial is the greatest degree of any term of the polynomial.

**EXAMPLE 2** Find the degree of each polynomial and tell whether the polynomial is a monomial, binomial, trinomial, or none of these.

\[ \begin{align*}
\text{a. } & -2t^2 + 3t + 6 \\
\text{b. } & 15x - 10 \\
\text{c. } & 7x + 3x^3 + 2x^2 - 1
\end{align*} \]

**Solution**

a. The degree of the trinomial \(-2t^2 + 3t + 6\) is 2, the greatest degree of any of its terms.

b. The degree of the binomial \(15x - 10\) or \(15x^1 - 10\) is 1.

c. The degree of the polynomial \(7x + 3x^3 + 2x^2 - 1\) is 3.

**PRACTICE**

Find the degree of each polynomial and tell whether the polynomial is a monomial, binomial, trinomial, or none of these.

\[ \begin{align*}
\text{a. } & 5b^2 - 3b + 7 \\
\text{b. } & 7t + 3 \\
\text{c. } & 5x^2 + 3x - 6x^3 + 4
\end{align*} \]
Example 3

Complete the table for the polynomial

\[ 7x^2y - 6xy + x^2 - 3y + 7 \]

Use the table to give the degree of the polynomial.

**Solution**

<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical Coefficient</th>
<th>Degree of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7x^2y)</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>(-6xy)</td>
<td>-6</td>
<td>2</td>
</tr>
<tr>
<td>(x^2)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(-3y)</td>
<td>-3</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

The degree of the polynomial is 3.

Practice 3

Complete the table for the polynomial \(-3x^3y^2 + 4xy^2 - y^2 + 3x - 2\).

<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical Coefficient</th>
<th>Degree of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3x^3y^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4xy^2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-y^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Objective

2 Evaluating Polynomials

Polynomials have different values depending on replacement values for the variables.

Example 4

Find the value of each polynomial when \(x = -2\).

a. \(-5x + 6\)  

\[ \begin{align*}
-5x + 6 &= -5(-2) + 6 \quad \text{Replace } x \text{ with } -2 \\
&= 10 + 6 \\
&= 16
\end{align*} \]

b. \(3x^2 - 2x + 1\)  

\[ \begin{align*}
3x^2 - 2x + 1 &= 3(-2)^2 - 2(-2) + 1 \quad \text{Replace } x \text{ with } -2 \\
&= 3(4) + 4 + 1 \\
&= 12 + 4 + 1 \\
&= 17
\end{align*} \]

Practice 4

Find the value of each polynomial when \(x = -3\).

a. \(-10x + 1\)  

b. \(2x^2 - 5x + 3\)

Many physical phenomena can be modeled by polynomials.
Section 5.2  Adding and Subtracting Polynomials

Finding the Height of a Dropped Object
The Swiss Re Building, in London, is a unique building. Londoners often refer to it as the “pickle building.” The building is 592.1 feet tall. An object is dropped from the highest point of this building. Neglecting air resistance, the height in feet of the object above ground at time \( t \) seconds is given by the polynomial \(-16t^2 + 592.1\). Find the height of the object when \( t = 1 \) second and when \( t = 6 \) seconds.

Solution  To find each height, we evaluate the polynomial when \( t = 1 \) and when \( t = 6 \).

\[
-16t^2 + 592.1 = -16(1)^2 + 592.1 \quad \text{Replace } t \text{ with 1.}
\]

\[
= -16(1) + 592.1 \quad = -16 + 592.1 \quad = 576.1
\]

The height of the object at 1 second is 576.1 feet.

\[
-16t^2 + 592.1 = -16(6)^2 + 592.1 \quad \text{Replace } t \text{ with 6.}
\]

\[
= -16(36) + 592.1 \quad = -576 + 592.1 = 16.1
\]

The height of the object at 6 seconds is 16.1 feet.

Practice 5
The cliff divers of Acapulco dive 130 feet into La Quebrada several times a day for the entertainment of the tourists. If a tourist is standing near the diving platform and drops his camera off the cliff, the height of the camera above the water at time \( t \) seconds is given by the polynomial \(-16t^2 + 130\). Find the height of the camera when \( t = 1 \) second and when \( t = 2 \) seconds.

Simplifying Polynomials by Combining Like Terms
Polynomials with like terms can be simplified by combining the like terms. Recall that like terms are terms that contain exactly the same variables raised to exactly the same powers.

<table>
<thead>
<tr>
<th>Like Terms</th>
<th>Unlike Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5x^2, -7x^2 )</td>
<td>( 3x, 3y )</td>
</tr>
<tr>
<td>( y, 2y )</td>
<td>( -2x^2, -5x )</td>
</tr>
<tr>
<td>( \frac{1}{2}a^2b, -a^2b )</td>
<td>( 6st^2, 4s^2t )</td>
</tr>
</tbody>
</table>

Only like terms can be combined. We combine like terms by applying the distributive property.

Example 6  Simplify each polynomial by combining any like terms.

a. \(-3x + 7x\)  
   Solution  \(-3x + 7x = (-3 + 7)x = 4x\)

b. \(x + 3x^2\)  

   These terms cannot be combined because \( x \) and \( 3x^2 \) are not like terms.

   c. \(9x^3 + x^3\)  
   Solution  \(9x^3 + x^3 = 9x^3 + 1x^3 = 10x^3\)

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\[
\text{d. } 11x^2 + 5 + 2x^2 - 7 = 11x^2 + 2x^2 + 5 - 7 \\
= 13x^2 - 2 \\
\text{Combine like terms.}
\]

\[
\text{e. } \frac{2}{5}x^4 + \frac{2}{3}x^3 - x^2 + \frac{1}{10}x^4 - \frac{1}{6}x^3 \\
= \left(\frac{2}{5} + \frac{1}{10}\right)x^4 + \left(\frac{2}{3} - \frac{1}{6}\right)x^3 - x^2 \\
= \left(\frac{4}{10} + \frac{1}{10}\right)x^4 + \left(\frac{4}{6} - \frac{1}{6}\right)x^3 - x^2 \\
= \frac{5}{10}x^4 + \frac{3}{6}x^3 - x^2 \\
= \frac{1}{2}x^4 + \frac{1}{2}x^3 - x^2
\]

**PRACTICE** Simplify each polynomial by combining any like terms.

a. \(-4y + 2y\)  
   b. \(z + 5x^3\)  
   c. \(15x^3 - x^3\)  
   d. \(7a^2 - 5 - 3a^2 - 7\)  
   e. \(\frac{3}{8}x^3 - x^2 + \frac{5}{6}x^4 + \frac{1}{12}x^3 - \frac{1}{2}x^4\)

**CONCEPT CHECK**

When combining like terms in the expression \(5x - 8x^2 - 8x\), which of the following is the proper result?

a. \(-11x^2\)  
   b. \(-8x^2 - 3x\)  
   c. \(-11x\)  
   d. \(-11x^4\)

**EXAMPLE 7** Combine like terms to simplify.

\[-9x^2 + 3xy - 5y^2 + 7yx\]

**Solution**

\[-9x^2 + 3xy - 5y^2 + 7yx = -9x^2 + (3 + 7)xy - 5y^2 \]

\[= -9x^2 + 10xy - 5y^2 \]

**EXAMPLE 8** Write a polynomial that describes the total area of the squares and rectangles shown below. Then simplify the polynomial.

**Solution**

\[
\text{Area: } x \cdot x + 3 \cdot x + 3 \cdot 3 + 4 \cdot 3 + x \cdot 2x \\
= x^2 + 3x + 9 + 4x + 2x^2 \\
= 3x^2 + 7x + 9
\]

**PRACTICE** Write a polynomial that describes the total area of the squares and rectangles shown below. Then simplify the polynomial.

Answer to Concept Check: b
We now practice adding and subtracting polynomials.

Adding Polynomials
To add polynomials, combine all like terms.

**Example 9** Add \((-2x^2 + 5x - 1)\) and \((-2x^2 + x + 3)\).

**Solution**
\[
(-2x^2 + 5x - 1) + (-2x^2 + x + 3) = -2x^2 + 5x - 1 - 2x^2 + x + 3
\]
\[
= (-2x^2 - 2x^2) + (5x + 1x) + (-1 + 3)
\]
\[
= -4x^2 + 6x + 2
\]

**Practice**
Add \((-3x^2 - 4x + 9)\) and \((2x^2 - 2x)\).

**Example 10** Add \((4x^3 - 6x^2 + 2x + 7)\) and \((5x^2 - 2x)\).

**Solution**
\[
(4x^3 - 6x^2 + 2x + 7) + (5x^2 - 2x) = 4x^3 - 6x^2 + 2x + 7 + 5x^2 - 2x
\]
\[
= 4x^3 + (-6x^2 + 5x^2) + (2x - 2x) + 7
\]
\[
= 4x^3 - x^2 + 7
\]

**Practice**
Add \((-3x^3 + 7x^2 + 3x - 4)\) and \((3x^2 - 9x)\)

Polynomials can be added vertically if we line up like terms underneath one another.

**Example 11** Add \((7y^3 - 2y^2 + 7)\) and \((6y^2 + 1)\) using the vertical format.

**Solution** Vertically line up like terms and add.
\[
\begin{align*}
7y^3 &- 2y^2 + 7 \\
-6y^2 &+ 1 \\
\hline
7y^3 &+ 4y^2 + 8
\end{align*}
\]

**Practice**
Add \((5z^3 + 3z^2 + 4z)\) and \((5z^2 + 4z)\) using the vertical format.

To subtract one polynomial from another, recall the definition of subtraction. To subtract a number, we add its opposite: \(a - b = a + (-b)\). To subtract a polynomial, we also add its opposite. Just as \(-b\) is the opposite of \(b\), \(-(x^2 + 5)\) is the opposite of \((x^2 + 5)\).

**Example 12** Subtract \((5x - 3) - (2x - 11)\)

**Solution** From the definition of subtraction, we have
\[
(5x - 3) - (2x - 11) = (5x - 3) + [- (2x - 11)]
\]
\[
= (5x - 3) + (-2x + 11)
\]
\[
= (5x - 2x) + (-3 + 11)
\]
\[
= 3x + 8
\]

**Practice**
Subtract \((8x - 7) - (3x - 6)\)
**Subtracting Polynomials**

To subtract two polynomials, change the signs of the terms of the polynomial being subtracted and then add.

**Example 13** Subtract: \((2x^3 + 8x^2 - 6x) - (2x^3 - x^2 + 1)\)

**Solution** Change the sign of each term of the second polynomial and then add.

\[
(2x^3 + 8x^2 - 6x) - (2x^3 - x^2 + 1) = (2x^3 + 8x^2 - 6x) + (-2x^3 + x^2 - 1) \\
= 2x^3 - 2x^3 + 8x^2 + x^2 - 6x - 1 \\
= 9x^2 - 6x - 1 \\
\]

Combine like terms.

**Example 14** Subtract \((5y^2 + 2y - 6)\) from \((-3y^2 - 2y + 11)\) using the vertical format.

**Solution** Arrange the polynomials in vertical format, lining up like terms.

\[
\begin{array}{c}
-3y^2 - 2y + 11 \\
\hline
-(5y^2 + 2y - 6) \\
\hline
-8y^2 - 4y + 17
\end{array}
\]

**Example 15** Subtract \((5z - 7)\) from the sum of \((8z + 11)\) and \((9z - 2)\).

**Solution** Notice that \((5z - 7)\) is to be subtracted from a sum. The translation is

\[
[(8z + 11) + (9z - 2)] - (5z - 7) \\
= 8z + 11 + 9z - 2 - 5z + 7 \\
= 8z + 9z - 5z + 11 - 2 + 7 \\
= 12z + 16 \\
\]

Combine like terms.

**Example 16** Add or subtract as indicated.

a. \((3x^2 - 6xy + 5y^2) + (-2x^2 + 8xy - y^2)\)

b. \((9a^2b^2 + 6ab - 3ab^3) - (5b^2a + 2ab - 3 - 9b^2)\)

**Solution**

a. \((3x^2 - 6xy + 5y^2) + (-2x^2 + 8xy - y^2) = 3x^2 - 6xy + 5y^2 - 2x^2 + 8xy - y^2 = x^2 + 2xy + 4y^2\) Combine like terms.

b. \((9a^2b^2 + 6ab - 3ab^3) - (5b^2a + 2ab - 3 - 9b^2) = 9a^2b^2 + 6ab - 3ab^3 - 5b^2a - 2ab + 3 + 9b^2 = 9a^2b^2 + 4ab - 8ab^2 + 3 + 9b^2\) Change the sign of each term of the polynomial being subtracted. Combine like terms.
Section 5.2  Adding and Subtracting Polynomials

Add or subtract as indicated.

a. \((3a^2 - 4ab + 7b^2) + (-8a^2 + 3ab - b^2)\)

b. \((5x^2y^2 - 6xy - 4xy^2) - (2x^2y^2 + 4xy - 5 + 6y^2)\)

Practice

16

Add or subtract as indicated.

a. \(1 + 3a^2 - 4ab + 7b^2\)

b. \((5x^2y^2 - 6xy - 4xy^2) - (2x^2y^2 + 4xy - 5 + 6y^2)\)

Answers to Concept Check:

a. \(3y\)  
b. \(2\)  
c. \(-3y\)  
d. \(2\)  
e. cannot be simplified

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Not all choices will be used.

least  
monomial  
trinomial  
coefficient  
greatest  
binomial  
constant

1. A _______ is a polynomial with exactly two terms.
2. A _______ is a polynomial with exactly one term.
3. A _______ is a polynomial with exactly three terms.
4. The numerical factor of a term is called the _______.
5. A number term is also called a _______.
6. The degree of a polynomial is the _______ degree of any term of the polynomial.

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.

1. Objective 1

2. Objective 2

3. Objective 3

4. Objective 4

7. For Example 2, why is the degree of each term found when the example asks for the degree of the polynomial only?

8. From Example 3, what does the value of a polynomial depend on?

9. When combining any like terms in a polynomial, as in Examples 4–6, what are we doing to the polynomial?

10. From Example 7, when we simply remove parentheses and combine the like terms of two polynomials, what operation do we perform? Is this true of Examples 9–11?

5.2 Exercise Set

Find the degree of each of the following polynomials and determine whether it is a monomial, binomial, trinomial, or none of these. See Examples 1 through 3.

1. \(x + 2\)  
2. \(-6y^2 + 4\)  
3. \(9m^3 - 5m^2 + 4m - 8\)  
4. \(a + 5a^2 + 3a^3 - 4a^4\)  
5. \(12x^3y - x^2y^2 - 12x^2y^4\)  
6. \(7r^2s^2 + 2rs - 3rs^5\)  
7. \(3 - 5x^8\)  
8. \(5y^7 + 2\)

In the second column, write the degree of the polynomial in the first column. See Examples 1 through 3.

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3xy^2 - 4)</td>
<td></td>
</tr>
<tr>
<td>(8x^4y^5)</td>
<td></td>
</tr>
<tr>
<td>(11)</td>
<td></td>
</tr>
<tr>
<td>(4z^6 + 3z^2)</td>
<td></td>
</tr>
</tbody>
</table>

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Find the value of each polynomial when (a) \( x = 0 \) and (b) \( x = -1 \). See Examples 4 and 5.

13. \( 5x - 6 \)
14. \( 2x - 10 \)
15. \( x^2 - 5x - 2 \)
16. \( x^2 + 3x - 4 \)
17. \( -x^3 + 4x^2 - 15x + 1 \)
18. \( -2x^3 + 3x^2 - 6x + 1 \)

The CN Tower in Toronto, Ontario, is 1821 feet tall and is the world’s tallest self-supporting structure. An object is dropped from the Skypod of the Tower, which is at 1150 feet. Neglecting air resistance, the height of the object at time \( t \) seconds is given by the polynomial \(-16t^2 + 1150\). Find the height of the object at the given times.

<table>
<thead>
<tr>
<th>Time, ( t ) (in seconds)</th>
<th>Height (-16t^2 + 1150)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.</td>
<td>1</td>
</tr>
<tr>
<td>20.</td>
<td>7</td>
</tr>
<tr>
<td>21.</td>
<td>3</td>
</tr>
<tr>
<td>22.</td>
<td>6</td>
</tr>
</tbody>
</table>

23. The polynomial \(-7.5x^2 + 103x + 2000\) models the yearly number of visitors (in thousands) \( x \) years after 2006 at Acadia National Park in Maine. Use this polynomial to estimate the number of visitors to the park in 2016.

24. The polynomial \(-0.13x^2 + x + 827\) models the yearly number of visitors (in thousand) \( x \) years after 2006 at Canyon De Chelly National Monument in Arizona. Use this polynomial to estimate the number of visitors to the park in 2010.

Simplify each of the following by combining like terms. See Examples 6 and 7.

25. \( 9x - 20x \)
26. \( 14y - 30y \)
27. \( 14x^2 + 9x^2 \)
28. \( 18x^3 - 4x^3 \)
29. \( 15x^2 - 3x^2 - y \)
30. \( 12k^4 - 9k^2 + 11 \)
31. \( 8s - 5x + 4s \)

32. \( 5y + 7y - 6y \)
33. \( 0.1y^2 - 1.2y^2 + 6.7 - 1.9 \)
34. \( 7.6y + 3.2y^2 - 8y - 2.5y^2 \)
35. \( \frac{2}{3}x^4 + 12x^3 + \frac{1}{6}x - 19x - 19 \)
36. \( \frac{2}{5}x^4 - 23x^2 + \frac{11}{15}x^4 + 5x^2 - 5 \)
37. \( \frac{5}{2}x^3 - \frac{3}{5}x^3 + x^2 - \frac{1}{4}x^3 + 6 \)
38. \( \frac{1}{6}x^4 - \frac{1}{7}x^2 + 5 \)
39. \( 6x^2 - 4ay + 7b^2 - a^2 - 5ab + 9b^2 \)
40. \( x^2y + xy - y + 10x^3y - 2y + xy \)

Perform the indicated operations. See Examples 9 through 13.

41. \((8 + 2a) - (-a - 3)\)
42. \((4 + 5a) - (-a - 5)\)
43. \((2x^2 + 5) - (3x^2 - 9)\)
44. \((5x^2 + 4) - (7x^2 - 6)\)
45. \((-7x + 5) + (-3x^2 + 7x + 5)\)
46. \((3x - 8) + (4x^2 - 3x + 3)\)
47. \(3x - (5x - 9)\)
48. \(4 - (-12y - 4)\)
49. \((2x^2 + 3x - 9) - (-4x + 7)\)
50. \((-7x^2 + 4x + 7) - (-8x + 2)\)

Perform the indicated operations. See Examples 11, 14, and 15.

51. \(3t^2 + 4 + 5t^2 - 8\)
52. \(7x^5 + 3 + 2x^3 + 1\)
53. \(4x^2 - 8z + 3 - (6x^2 + 8z - 3)\)
54. \(7a^2 - 9a + 6 - (11a^2 - 4a + 2)\)
55. \(5x^3 - 4x^2 + 6x - 2 - (3x^3 - 2x^2 - x - 4)\)
56. \(5u^5 - 4u^3 + 3u - 7 - (3u^5 + 6u^3 - 8u + 2)\)

57. Subtract \((19x^2 + 5)\) from \((81x^2 + 10)\).
58. Subtract \((2x + xy)\) from \((3x - 9xy)\).
59. Subtract \((2x + 2)\) from the sum of \((8x + 1)\) and \((6x + 3)\).
60. Subtract \((-12x - 3)\) from the sum of \((-5x - 7)\) and \((12x + 3)\).

MIXED PRACTICE

Perform the indicated operations.

61. \((2y + 20) + (5y - 30)\)
62. \((14y + 12) + (-3y - 5)\)
63. \((x^2 + 2x + 1) - (3x^2 - 6x + 2)\)
64. \((5y^2 - 3y - 1) - (2y^2 + y + 1)\)
65. \((3x^2 + 5x - 8) + (5x^2 + 9x + 12) - (8x^2 - 14)\)
66. \((2x^2 + 7x - 9) + (x^2 - x + 10) - (3x^2 - 30)\)
67. \((-a^2 + 1) - (a^2 - 3) + (5a^2 - 6a + 7)\)
68. \((-m^2 + 3) - (m^2 - 13) + (6m^2 - m + 1)\)
Perform each indicated operation.
69. Subtract 4x from \((7x - 3)\).
70. Subtract y from \((x^2 - 4y + 1)\).
71. Add \((4x^2 - 6x + 1)\) and \((3x^2 + 2x + 1)\).
72. Add \((-3x^2 - 5x + 2)\) and \((x^2 - 6x + 9)\).
73. Subtract \((5x + 7)\) from \((7x^2 + 3x + 9)\).
74. Subtract \((5y^2 + 8y + 2)\) from \((7y^2 + 9y - 8)\).
75. Subtract \((4y^2 - 6y - 3)\) from the sum of \((8y^2 + 7)\) and \((6y + 9)\).
76. Subtract \((4x^2 - 2x + 2)\) from the sum of \((x^2 + 7x + 1)\) and \((7x + 5)\).

Find the area of each figure. Write a polynomial that describes the total area of the rectangles and squares shown in Exercises 77 and 78. Then simplify the polynomial. See Example 8.

Add or subtract as indicated. See Example 16.
79. \((9a + 6b - 5) + (-11a - 7b + 6)\)
80. \((3x - 2 + 6y) + (7x - 2 - y)\)
81. \((4x^2 + 3x^2 + 3) - (x^2 + y^2 - 2)\)
82. \((7a^2 - 3b^2 + 10) - (-2a^2 + b^2 - 12)\)
83. \((x^2 + 2xy - y^2) + (5x^2 - 4xy + 20y^2)\)
84. \((a^2 - ab + 4b^2) + (6a^2 + 8ab - b^2)\)
85. \((11r^2 + 16rs - 3 - 2r^2s^2) - (3r^2 + 5 - 9r^2s^2)\)
86. \((3x^2y - 6xyz + x^2y^2 - 5) - (11x^2y^2 - 1 + 5xy^2)\)

Simplify each polynomial by combining like terms.
87. \(7.75x + 9.16x^2 - 1.27 - 14.58x^2 - 18.34\)
88. \(1.85x^2 - 3.76x + 9.25x^2 + 10.76 - 4.21x\)

Perform each indicated operation.
89. \([ (7.9y^4 - 6.8y^3 + 3.3y) + (6.1y^3 - 5) ] - (4.2y^4 + 1.1y - 1)\)
90. \([ (1.2x^2 - 3x + 9.1) - (7.8x^2 - 3.1 + 8) ] + (1.2x - 6)\)

**CONCEPT EXTENSIONS**
Recall that the perimeter of a figure is the sum of the lengths of its sides. For Exercises 97 through 100, find the perimeter of each figure. Write the perimeter as a simplified polynomial.

**Exercise:**

97. \(\triangle\) perimeter is \(9x - 2x\) feet, \(7x\) feet, and \(10x\) feet.
98. \(\triangle\) perimeter is \(3x\) centimeters, \(5x\) centimeters, and \((4x - 1)\) feet.
99. \(\triangle\) perimeter is \((2x^2 + 5)\) feet, \((-x^2 + 3x)\) feet, and \((4x + 5)\) feet.
100. \(\triangle\) perimeter is \((x^2 - 6x - 2)\) centimeters, \((x^2 - 4)\) centimeters, and \(5x\) centimeters.
101. A wooden beam is \((4y^2 + 4y + 1)\) meters long. If a piece \((y^2 - 10)\) meters is cut, express the length of the remaining piece of beam as a polynomial in \(y\).
102. A piece of quarter-round molding is \((13x - 7)\) inches long. If a piece \((2x + 2)\) inches is removed, express the length of the remaining piece of molding as a polynomial in \(x\).
The number of worldwide Internet users (in millions) x years after the year 2000 is given by the polynomial $4.8x^3 + 104x + 431$ for the years 1995 through 2015. Use this polynomial for Exercises 103 and 104.

103. Estimate the number of Internet users in the world in 2015.

104. Use the given polynomial to predict the number of Internet users in the world in 2020.

CONCEPT EXTENSIONS

105. Describe how to find the degree of a term.

106. Describe how to find the degree of a polynomial.

107. Explain why $xyz$ is a monomial while $x + y + z$ is a trinomial.

108. Explain why the degree of the term $5y^3$ is 3 and the degree of the polynomial $2y + y + 2y$ is 1.

Match each expression on the left with its simplification on the right. Not all letters on the right must be used, and a letter may be used more than once.

109. $10y - 6y^2 - y$  
   A. $3y$

110. $5x + 5x$  
    B. $9y - 6y^2$

111. $(5x - 3) + (5x - 3)$  
    C. $10x$

112. $(15x - 3) - (5x - 3)$  
    D. $25x^2$

   E. $10x - 6$

   F. none of these

Simplify each expression by performing the indicated operation. Explain how you arrived at each answer. See the last Concept Check in this section.

113. a. $z + 3z$ b. $z \cdot 3z$
    c. $-z - 3z$ d. $(-z)(-3z)$

114. a. $5y + y$ b. $5y \cdot y$
    c. $-5y - y$ d. $(-5y)(-y)$

115. a. $m \cdot m \cdot m$ b. $m + m + m$
    c. $(-m)(-m)(-m)$ d. $-m - m - m$

116. a. $x + x$ b. $x \cdot x$
    c. $-x - x$ d. $(-x)(-x)$

Fill in the squares so that each is a true statement.

117. $3x^2 + 4x^2 = 7x^2$

118. $9y^7 + 3y^7 = 12y^7$

119. $2x^4 + 3x^4 - 5x^4 + 4x^4 = 6x^4 - 2x^4$

120. $3y^7 + 7y^7 - 2y^7 - y^7 = 10y^7 - 3y^7$

Write a polynomial that describes the surface area of each figure. (Recall that the surface area of a solid is the sum of the areas of the faces or sides of the solid.)

121.

122.

5.3 Multiplying Polynomials

OBJECTIVES

1 Multiply Monomials.
2 Use the Distributive Property to Multiply Polynomials.
3 Multiply Polynomials Vertically.

OBJECTIVE 1 Multiplying Monomials

Recall from Section 5.1 that to multiply two monomials such as $(-5x^3)$ and $(-2x^4)$, we use the associative and commutative properties and regroup. Remember, also, that to multiply exponential expressions with a common base, we use the product rule for exponents and add exponents.

$$(-5x^3)(-2x^4) = (-5)(-2)(x^3)(x^4) = 10x^7$$
EXAMPIES  Multiply.

1. \(6x \cdot 4x = (6 \cdot 4) (x \cdot x)\)  Use the commutative and associative properties.
   \[= 24x^2\]  Multiply.
2. \(-7x^2 \cdot 0.2x^5 = (-7 \cdot 0.2) (x^2 \cdot x^5)\)
   \[= -1.4x^7\]
3. \(\left(\frac{-1}{3}x^5\right) \left(-\frac{2}{9}x\right) = \left(\frac{-1}{3} \cdot -\frac{2}{9}\right) (x^5 \cdot x)\)
   \[= \frac{2}{27}x^6\]

PRACTICE  Multiply.

1. \(5y \cdot 2y\) \hspace{1cm} 2. \((5z^3) \cdot (-0.4z^5)\) \hspace{1cm} 3. \((-\frac{1}{9}b^6) \left(-\frac{7}{8}b^3\right)\)

✔ CONCEPT CHECK

Simplify.

a. \(3x \cdot 2x\) \hspace{1cm} b. \(3x + 2x\)

OBJECTIVE 2 Using the Distributive Property to Multiply Polynomials

To multiply polynomials that are not monomials, use the distributive property.

EXAMPLE 4 Use the distributive property to find each product.

a. \(5x(2x^3 + 6)\)

Solution

\[a. \quad 5x(2x^3 + 6) = 5x(2x^3) + 5x(6)\]  Use the distributive property.
\[= 10x^4 + 30x\]  Multiply.

b. \(-3x^2(5x^2 + 6x - 1)\)

\[= (-3x^2)(5x^2) + (-3x^2)(6x) + (-3x^2)(-1)\]  Use the distributive property. Multiply.
\[= -15x^4 - 18x^3 + 3x^2\]

PRACTICE  Use the distributive property to find each product.

a. \(3x(9x^5 + 11)\) \hspace{1cm} b. \(-6x^3(2x^2 - 9x + 2)\)

We also use the distributive property to multiply two binomials. To multiply \((x + 3)\) by \((x + 1)\), distribute the factor \((x + 3)\) first.

\[(x + 3)(x + 1) = x(x + 1) + 3(x + 1)\]  Distribute \((x + 3)\).
\[= x(x) + x(1) + 3(x) + 3(1)\]  Apply the distributive property a second time. Multiply.
\[= x^2 + x + 3x + 3\]  Combine like terms.

Answers to Concept Check:

a. \(6x^2\) \hspace{1cm} b. \(5x\)

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This idea can be expanded so that we can multiply any two polynomials.

### To Multiply Two Polynomials

Multiply each term of the first polynomial by each term of the second polynomial and then combine like terms.

#### Example 5

Multiply: 
\[(3x + 2)(2x - 5)\]

**Solution**

Multiply each term of the first binomial by each term of the second.

\[
(3x + 2)(2x - 5) = 3x(2x) + 3x(-5) + 2(2x) + 2(-5) = 6x^2 - 15x + 4x - 10
\]

Multiply.

\[= 6x^2 - 11x - 10\]

Combine like terms.

**Practice**

5. Multiply: 
\[(5x - 2)(2x + 3)\]

#### Example 6

Multiply: 
\[(2x - y)^2\]

**Solution**

Recall that \(a^2 = a \cdot a\), so 
\[(2x - y)^2 = (2x - y)(2x - y)\]. Multiply each term of the first polynomial by each term of the second.

\[
(2x - y)(2x - y) = 2x(2x) + 2x(-y) + (-y)(2x) + (-y)(-y) = 4x^2 - 2xy - 2xy + y^2
\]

Multiply.

\[= 4x^2 - 4xy + y^2\]

Combine like terms.

**Practice**

6. Multiply: 
\[(5x - 3y)^2\]

**Concept Check**

Square where indicated. Simplify if possible.

a. \((4a)^2 + (3b)^2\)

b. \((4a + 3b)^2\)

#### Example 7

Multiply: 
\[(t + 2)(3t^2 - 4t + 2)\]

**Solution**

Multiply each term of the first polynomial by each term of the second.

\[
(t + 2)(3t^2 - 4t + 2) = t(3t^2) + t(-4t) + t(2) + 3(3t^2) + 2(-4t) + 2(2)
\]

\[= 3t^3 - 4t^2 + 2t + 3t^2 - 8t + 4\]

Combine like terms.

**Practice**

7. Multiply: 
\[(y + 4)(2y^2 - 3y + 5)\]

#### Example 8

Multiply: 
\[(3a + b)^3\]

**Solution**

Write \((3a + b)^3\) as \((3a + b)(3a + b)(3a + b)\).

\[
(3a + b)(3a + b)(3a + b) = (9a^2 + 3ab + 3ab + b^2)(3a + b)
\]

\[= (9a^2 + 6ab + b^2)(3a + b)\]

\[= 9a^2(3a + b) + 6ab(3a + b) + b^2(3a + b)\]

\[= 27a^3 + 9a^2b + 18ab^2 + 6ab^2 + 3ab^2 + b^3\]

\[= 27a^3 + 27ab^2 + 9ab^2 + b^3\]

**Practice**

8. Multiply: 
\[(s + 2t)^3\]

Answers to Concept Check:

a. \(16a^2 + 9b^2\)

b. \(16a^2 + 24ab + 9b^2\)

**Practice**

8. Multiply: 
\[(s + 2t)^3\]
Another convenient method for multiplying polynomials is to use a vertical format similar to the format used to multiply real numbers. We demonstrate this method by multiplying \((3y^2 - 4y + 1)\) by \((y + 2)\).

**Example 9** Multiply \((3y^2 - 4y + 1)\) by \((y + 2)\). Use a vertical format.

**Solution**

\[
\begin{array}{c}
3y^2 - 4y + 1 \\
\times \quad y + 2
\end{array}
\]

1st, multiply \(3y^2 - 4y + 1\) by 2.

\[
6y^2 - 8y + 2
\]

2nd, multiply \(3y^2 - 4y + 1\) by \(y\).

\[
3y^3 - 4y^2 + y
\]

3rd, combine like terms.

Thus, \((3y^2 - 4y + 1)(y + 2) = 3y^3 + 2y^2 - 7y + 2\).

When multiplying vertically, be careful if a power is missing; you may want to leave space in the partial products and take care that like terms are lined up.

**Example 10** Multiply \((2x^3 - 3x + 4)\) by \((x^2 + 1)\). Use a vertical format.

**Solution**

\[
\begin{array}{c}
2x^3 - 3x + 4 \\
\times \quad x^2 + 1
\end{array}
\]

Leave space for missing powers of \(x\).

\[
2x^5 - 3x^3 + 4x^2
\]

Line up terms.

\[
2x^3 - x^3 + 4x^2 - 3x + 4
\]

Combine like terms.

**Practice**

 Multiply \((5x^2 - 3x + 5)\) by \((x - 4)\). Use a vertical format.

**Example 11** Find the product of \((2x^2 - 3x + 4)\) and \((x^2 + 5x - 2)\) using a vertical format.

**Solution** First, we arrange the polynomials in a vertical format. Then we multiply each term of the second polynomial by each term of the first polynomial.

\[
\begin{array}{c}
2x^2 - 3x + 4 \\
\times \quad x^2 + 5x - 2
\end{array}
\]

\[
-4x^2 + 6x - 8
\]

\[
10x^3 - 15x^2 + 20x
\]

\[
2x^4 - 3x^3 + 4x^2
\]

\[
2x^4 + 7x^3 - 15x^2 + 26x - 8
\]

Multiply \(2x^2 - 3x + 4\) by \(-2\).

Multiply \(2x^2 - 3x + 4\) by \(5x\).

Multiply \(2x^2 - 3x + 4\) by \(x^2\).

Combine like terms.

**Practice**

Find the product of \((5x^2 + 2x - 2)\) and \((x^2 - x + 3)\) using a vertical format.
Fill in each blank with the correct choice.

1. The expression $5x(3x + 2)$ equals $5x \cdot 3x + 5x \cdot 2$ by the ____________ property.
   a. commutative   b. associative   c. distributive

2. The expression $(x + 4)(7x - 1)$ equals $x(7x - 1) + 4(7x - 1)$ by the ____________ property.
   a. commutative   b. associative   c. distributive

3. The expression $(5y - 1)^2$ equals ____________.
   a. $2(5y - 1)$   b. $(5y - 1)(5y + 1)$   c. $(5y - 1)(5y - 1)$

4. The expression $9x \cdot 3x$ equals ____________.
   a. $27x$   b. $27x^2$   c. $12x$   d. $12x^2$

Multiplying. See Examples 1 through 3.

1. $-4n^3 \cdot 7n^2$
2. $9y^6(-3y^3)$
3. $(-3.1x^3)(4x^9)$
4. $(-5.2x^4)(3x^4)$
5. $\left(\frac{1}{3}x^2\right) \left(\frac{2}{5}y^3\right)$
6. $\left(-\frac{3}{4}x^2\right) \left(\frac{1}{7}y^4\right)$
7. $(2x)(-3x^2)(4x^5)$
8. $(x)(5x^4)(-6x^7)$

Multiplying. See Example 4.

9. $3x(2x + 5)$
10. $2x(6x + 3)$
11. $-2a(a + 4)$
12. $-3a(2a + 7)$
13. $3x(2x^2 - 3x + 4)$
14. $4x(5x^2 - 6x - 10)$
15. $-2a^2(3a^2 - 2a + 3)$
16. $-4b^2(3b^3 - 12b^2 - 6)$
17. $-y(4x^3 - 7x^2y + xy^2 + 3y^3)$
18. $-x(6y^3 - 5xy^2 + x^2y - 5x^3)$
19. $\frac{1}{2}x^2(8x^2 - 6x + 1)$
20. $\frac{1}{3}y^2(9y^2 - 6y + 1)$

Multiplying. See Examples 5 and 6.

21. $(x + 4)(x + 3)$
22. $(x + 2)(x + 9)$
23. $(a + 7)(a - 2)$
24. $(y - 10)(y + 11)$
25. $(x + \frac{2}{3})(x - \frac{1}{3})$
26. $(x + \frac{3}{5})(x - \frac{2}{5})$
27. $(3x^2 + 1)(4x^2 + 7)$
28. $(5x^2 + 2)(6x^2 + 2)$
29. $(2y - 4)^2$
30. $(6x - 7)^2$
31. $(4x - 3)(3x - 5)$
32. $(8x - 3)(2x - 4)$
33. $(3x^2 + 1)^2$
34. $(x^2 + 4)^2$
35. Perform the indicated operations.
   a. $4y^2(-y^2)$
   b. $4y^2 - y^2$
   c. Explain the difference between the two expressions.
36. Perform the indicated operations.
   a. $9x^2(-10x^2)$
   b. $9x^2 - 10x^2$
   c. Explain the difference between the two expressions.

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In this section, we review operations on monomials. Study the box below, then proceed. See Sections 2.1, 5.1, and 5.2. (Continued on next page)

### Operations on Monomials

<table>
<thead>
<tr>
<th>Operation</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply</td>
<td>Review the product rule for exponents.</td>
</tr>
<tr>
<td>Divide</td>
<td>Review the quotient rule for exponents.</td>
</tr>
<tr>
<td>Add or Subtract</td>
<td>Remember, we may only combine like terms.</td>
</tr>
</tbody>
</table>

---

**Multiplying Polynomials**

1. **Multiply.** See Example 7
   - 37. \((x - 2)(x^2 - 3x + 7)\)
   - 38. \((x + 3)(x^2 + 5x - 8)\)
   - 39. \((x + 5)(x^2 - 3x + 4)\)
   - 40. \((a + 2)(a^3 - 3a^2 + 7)\)
   - 41. \((2a - 3)(5a^2 - 6a + 4)\)
   - 42. \((3 + h)(2 - 5h - 3h^2)\)

2. **Multiply.** See Example 8
   - 43. \((x + 2)^3\)
   - 44. \((y - 1)^3\)
   - 45. \((2y - 3)^3\)
   - 46. \((3x + 4)^3\)

3. **Multiply vertically.** See Examples 9 through 11
   - 47. \((2x - 11)(6x + 1)\)
   - 48. \((4x - 7)(5x + 1)\)
   - 49. \((5x + 1)(2x^2 + 4x - 1)\)
   - 50. \((4x - 5)(8x^2 + 2x - 4)\)
   - 51. \((x^2 + 5x - 7)(2x^2 - 7x - 9)\)
   - 52. \((3x^2 - x + 2)(x^2 + 2x + 1)\)

---

**Mixed Practice**

1. **Multiply.** See Examples 1 through 11
   - 53. \(-1.2y(-7y^6)\)
   - 54. \(-4.2x(-2x^5)\)
   - 55. \(-3x(x^2 + 2x - 8)\)
   - 56. \(-5x(x^2 - 3x + 10)\)
   - 57. \((x + 19)(2x + 1)\)
   - 58. \((3y + 4)(y + 11)\)
   - 59. \(\left(x + \frac{1}{7}\right)\left(x - \frac{3}{7}\right)\)
   - 60. \(\left(m + \frac{2}{9}\right)\left(m - \frac{1}{9}\right)\)
   - 61. \(3y + 5)^2\)
   - 62. \(7y + 2)^2\)
   - 63. \((a + 4)(a^2 - 6a + 6)\)
   - 64. \((t + 3)(t^2 - 5t + 5)\)

---

**Review and Preview**

In this section, we review operations on monomials. Study the box below, then proceed. See Sections 2.1, 5.1, and 5.2. (Continued on next page)
Perform the operations on the monomials if possible. The first two rows have been completed for you.

<table>
<thead>
<tr>
<th>Monomials</th>
<th>Add</th>
<th>Subtract</th>
<th>Multiply</th>
<th>Divide</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6x, 3x$</td>
<td>$6x + 3x = 9x$</td>
<td>$6x - 3x = 3x$</td>
<td>$6x \cdot 3x = 18x^2$</td>
<td>$\frac{6x}{3x} = 2$</td>
</tr>
<tr>
<td>$-12x^2, 2x$</td>
<td>$-12x^2 + 2x$, can’t be simplified</td>
<td>$-12x^2 - 2x$, can’t be simplified</td>
<td>$-12x^2 \cdot 2x = -24x^3$</td>
<td>$\frac{-12x^2}{2x} = -6x$</td>
</tr>
<tr>
<td>75. $5a, 15a$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>76. $4y^7, 4y^3$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>77. $-3y^5, 9y^4$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>78. $-14x^2, 2x^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Concept Extensions**

79. Perform each indicated operation. Explain the difference between the two expressions.
   a. $(3x + 5) + (3x + 7)$
   b. $(3x + 5)(3x + 7)$

80. Perform each indicated operation. Explain the difference between the two expressions.
   a. $(8x - 3) - (5x - 2)$
   b. $(8x - 3)(5x - 2)$

**Mixed Practice**

Perform the indicated operations. See Sections 5.2 and 5.3.

81. $(3x - 1) + (10x - 6)$
82. $(2x - 1) + (10x - 7)$
83. $(3x - 1)(10x - 6)$
84. $(2x - 1)(10x - 7)$
85. $(3x - 1) - (10x - 6)$
86. $(2x - 1) - (10x - 7)$

**Concept Extensions**

87. The area of the larger rectangle on the right is $x(x + 3)$. Find another expression for this area by finding the sum of the areas of the two smaller rectangles.

88. Write an expression for the area of the larger rectangle on the right in two different ways.

89. The area of the figure on the right is $(x + 2)(x + 3)$. Find another expression for this area by finding the sum of the areas of the four smaller rectangles.

90. Write an expression for the area of the figure in two different ways.

Simplify. See the Concept Checks in this section.

91. $5a + 6a$
92. $5a \cdot 6a$

Square where indicated. Simplify if possible.

93. $(5x)^2 + (2y)^2$
94. $(5x + 2y)^2$

95. Multiply each of the following polynomials.
   a. $(a + b)(a - b)$
   b. $(2x + 3y)(2x - 3y)$
   c. $(4x + 7)(4x - 7)$
   d. Can you make a general statement about all products of the form $(x + y)(x - y)$?

96. Evaluate each of the following.
   a. $(2 + 3)^2; 2^2 + 3^2$
   b. $(8 + 10)^2; 8^2 + 10^2$
   c. Does $(a + b)^2 = a^2 + b^2$ no matter what the values of $a$ and $b$ are? Why or why not?

97. Write a polynomial that describes the area of the shaded region. (Find the area of the larger square minus the area of the smaller square.)

98. Write a polynomial that describes the area of the shaded region. (See Exercise 97.)
Section 5.4  Special Products

OBJECTIVES
1. Multiply Two Binomials Using the FOIL Method.
2. Square a Binomial.
3. Multiply the Sum and Difference of Two Terms.
4. Use Special Products to Multiply Binomials.

OBJECTIVE 1 Using the FOIL Method

In this section, we multiply binomials using special products. First, a special order for multiplying binomials called the FOIL order or method is introduced. This method is demonstrated by multiplying \((3x + 1)(2x + 5)\) as shown below.

**The FOIL Method**

F stands for the product of the **First** terms. \((3x + 1)(2x + 5)\)

\(3x \times 2x = 6x^2\)  \(F\)

O stands for the product of the **Outer** terms. \((3x + 1)(2x + 5)\)

\((3x)(5) = 15x\)  \(O\)

I stands for the product of the **Inner** terms. \((3x + 1)(2x + 5)\)

\((1)(2x) = 2x\)  \(I\)

L stands for the product of the **Last** terms. \((3x + 1)(2x + 5)\)

\((1)(5) = 5\)  \(L\)

\[\begin{array}{cccc}
F & O & I & L \\
(3x + 1)(2x + 5) &= 6x^2 + 15x + 2x + 5 \\
&= 6x^2 + 17x + 5 & \text{Combine like terms.}
\end{array}\]

**CONCEPT CHECK**

Multiply \((3x + 1)(2x + 5)\) using methods from the last section. Show that the product is still \(6x^2 + 17x + 5\).

**EXAMPLE 1**

Multiply \((x - 3)(x + 4)\) by the FOIL method.

**Solution**

\[\begin{array}{cccc}
F & L & O & I \\
(x - 3)(x + 4) &= (x)(x) + (x)(4) + (-3)(x) + (-3)(4) \\
&= x^2 + 4x - 3x - 12 \\
&= x^2 + x - 12 & \text{Combine like terms.}
\end{array}\]

**EXAMPLE 2**

Multiply \((5x - 7)(x - 2)\) by the FOIL method.

**Solution**

\[\begin{array}{cccc}
F & L & O & I \\
(5x - 7)(x - 2) &= 5x(x) + 5x(-2) + (-7)(x) + (-7)(-2) \\
&= 5x^2 - 10x - 7x + 14 \\
&= 5x^2 - 17x + 14 & \text{Combine like terms.}
\end{array}\]

**PRACTICE**

1. Multiply \((x + 2)(x - 5)\) by the FOIL method.

2. Multiply \((4x - 9)(x - 1)\) by the FOIL method.

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**EXAMPLE 3** Multiply: \(2(y + 6)(2y - 1)\)

**Solution**

\[
2(y + 6)(2y - 1) = 2(2y^2 - 1y + 12y - 6)
\]

\[
= 2(2y^2 + 11y - 6)
\]

Simplify inside parentheses.

\[
= 4y^2 + 22y - 12
\]

Now use the distributive property.

**PRACTICE**

Multiply: \(3(x + 5)(3x - 1)\)

---

**OBJECTIVE**

Squaring Binomials

Now, try squaring a binomial using the FOIL method.

**EXAMPLE 4** Multiply: \((3y + 1)^2\)

**Solution**

\[
(3y + 1)^2 = (3y + 1)(3y + 1)
\]

\[
= (3y)(3y) + (3y)(1) + 1(3y) + 1(1)
\]

\[
= 9y^2 + 3y + 3y + 1
\]

\[
= 9y^2 + 6y + 1
\]

**PRACTICE**

Multiply: \((4x - 1)^2\)

Notice the pattern that appears in Example 4.

\[
(3y + 1)^2 = 9y^2 + 6y + 1
\]

\[
9y^2 \text{ is the first term of the binomial squared. } (3y)^2 = 9y^2.
\]

\[
6y \text{ is } 2 \text{ times the product of both terms of the binomial. } 2(3y)(1) = 6y.
\]

\[
1 \text{ is the second term of the binomial squared. } (1)^2 = 1.
\]

This pattern leads to the following, which can be used when squaring a binomial. We call these **special products**.

**Squaring a Binomial**

A binomial squared is equal to the square of the first term plus or minus twice the product of both terms plus the square of the second term.

\[
(a + b)^2 = a^2 + 2ab + b^2 \quad (a - b)^2 = a^2 - 2ab + b^2
\]

This product can be visualized geometrically.

The area of the large square is \( \text{side} \cdot \text{side} \).

\[
\text{Area} = (a + b)(a + b) = (a + b)^2
\]

The area of the large square is also the sum of the areas of the smaller rectangles.

\[
\text{Area} = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2
\]

Thus, \((a + b)^2 = a^2 + 2ab + b^2\).
### Example 5

Use a special product to square each binomial.

a. \((t + 2)^2\)  
   b. \((p - q)^2\)  
   c. \((2x + 5)^2\)  
   d. \((x^2 - 7y)^2\)

**Solution**

<table>
<thead>
<tr>
<th>First term squared</th>
<th>Plus or minus</th>
<th>Twice the product of the terms</th>
<th>Plus</th>
<th>Second term squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t^2)</td>
<td>+</td>
<td>(2t(2))</td>
<td>+</td>
<td>(4)</td>
</tr>
<tr>
<td>(p^2)</td>
<td>-</td>
<td>(2p(q))</td>
<td>+</td>
<td>(q^2)</td>
</tr>
<tr>
<td>((2x)^2)</td>
<td>+</td>
<td>(2(2x)(5))</td>
<td>+</td>
<td>(25)</td>
</tr>
<tr>
<td>((x^2)^2)</td>
<td>-</td>
<td>(2(x^2)(7y))</td>
<td>+</td>
<td>(49y^2)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
(t + 2)^2 &= t^2 + 4t + 4 \\
(p - q)^2 &= p^2 - 2pq + q^2 \\
(2x + 5)^2 &= 4x^2 + 20x + 25 \\
(x^2 - 7y)^2 &= x^4 - 14x^2y + 49y^2
\end{align*}
\]

### Practice 5

Use a special product to square each binomial.

a. \((b + 3)^2\)  
   b. \((x - y)^2\)  
   c. \((3y + 2)^2\)  
   d. \((a^2 - 5b)^2\)

**Helpful Hint**

Notice that

\[
(a + b)^2 \neq a^2 + b^2 \quad \text{The middle term } 2ab \text{ is missing.}
\]

\[
(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2
\]

Likewise,

\[
(a - b)^2 \neq a^2 - b^2
\]

\[
(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2
\]

### Objective 3

**Multiplying the Sum and Difference of Two Terms**

Another special product is the product of the sum and difference of the same two terms, such as \((x + y)(x - y)\). Finding this product by the FOIL method, we see a pattern emerge.

\[
\begin{align*}
(x + y)(x - y) &= x^2 - xy + xy - y^2 \\
&= x^2 - y^2
\end{align*}
\]

Notice that the middle two terms subtract out. This is because the Outer product is the opposite of the Inner product. Only the difference of squares remains.

**Multiplying the Sum and Difference of Two Terms**

The product of the sum and difference of two terms is the square of the first term minus the square of the second term.

\[
(a + b)(a - b) = a^2 - b^2
\]
EXAMPLE 6 Use a special product to multiply.

a. \(4(x + 4)(x - 4)\)

b. \((6t + 7)(6t - 7)\)

c. \(\left(x - \frac{1}{4}\right)\left(x + \frac{1}{4}\right)\)

d. \((2p - q)(2p + q)\)

e. \((3x^2 - 5y)(3x^2 + 5y)\)

Solution

- a. \(4(x + 4)(x - 4) = 4(x^2 - 4) = 4x^2 - 16\)
- b. \((6t + 7)(6t - 7) = (6t)^2 - 7^2 = 36t^2 - 49\)
- c. \(\left(x - \frac{1}{4}\right)\left(x + \frac{1}{4}\right) = x^2 - \left(\frac{1}{4}\right)^2 = x^2 - \frac{1}{16}\)
- d. \((2p - q)(2p + q) = (2p)^2 - q^2 = 4p^2 - q^2\)
- e. \((3x^2 - 5y)(3x^2 + 5y) = (3x^2)^2 - (5y)^2 = 9x^4 - 25y^2\)

PRACTICE Use a special product to multiply.

a. \(3(x + 5)(x - 5)\)

b. \((4b - 3)(4b + 3)\)

c. \(\left(x + \frac{2}{3}\right)\left(x - \frac{2}{3}\right)\)

d. \((5s + t)(5s - t)\)

e. \((2y - 3z^2)(2y + 3z^2)\)

CONCEPT CHECK

Match expression number 1 and number 2 to the equivalent expression or expressions in the list below.

1. \((a + b)^2\)
2. \((a + b)(a - b)\)
   - A. \((a + b)(a + b)\)
   - B. \(a^2 - b^2\)
   - C. \(a^2 + b^2\)
   - D. \(a^2 - 2ab + b^2\)
   - E. \(a^2 + 2ab + b^2\)

OBJECTIVE

Using Special Products

Let’s now practice multiplying polynomials in general. If possible, use a special product.

EXAMPLE 7 Use a special product, if possible.

a. \((x - 5)(3x + 4)\)

b. \((7x + 4)^2\)

c. \((y - 0.6)(y + 0.6)\)

d. \(\left(y + \frac{2}{5}\right)\left(3y^2 - \frac{1}{5}\right)\)

e. \((a - 3)(a^2 + 2a - 1)\)

Solution

- a. \((x - 5)(3x + 4) = 3x^2 + 4x - 15x - 20 = 3x^2 - 11x - 20\) FOIL.
- b. \((7x + 4)^2 = (7x)^2 + 2(7x)(4) + 4^2 = 49x^2 + 56x + 16\) Squaring a binomial.
- c. \((y - 0.6)(y + 0.6) = y^2 - (0.6)^2 = y^2 - 0.36\) Multiplying the sum and difference of 2 terms.
- d. \(\left(y + \frac{2}{5}\right)\left(3y^2 - \frac{1}{5}\right) = 3y^3 - \frac{1}{5}y^4 + \frac{6}{5}y^2 - \frac{2}{25}\) FOIL.
- e. I’ve inserted this product as a reminder that since it is not a binomial times a binomial, the FOIL order may not be used.

Answers to Concept Check:
1. A and E
2. B

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\[(a - 3)(a^2 + 2a - 1) = a(a^2 + 2a - 1) - 3(a^2 + 2a - 1)\]
\[= a^3 + 2a^2 - a - 3a^2 - 6a + 3\]
\[= a^3 - a^2 - 7a + 3\]

**PRACTICE**

7. Use a special product to multiply, if possible.
   a. \((4x + 3)(x - 6)\)
   b. \((7b - 2)^2\)
   c. \((x + 0.4)(x - 0.4)\)
   d. \((x^2 - \frac{3}{7})(3x^4 + \frac{2}{7})\)
   e. \((x + 1)(x^2 + 5x - 2)\)

**Vocabulary, Readiness & Video Check**

Answer each exercise true or false.

1. \((x + 4)^2 = x^2 + 16\)
2. For \((x + 6)(2x - 1)\), the product of the first terms is \(2x^2\).
3. \((x + 4)(x - 4) = x^2 + 16\)
4. The product \((x - 1)(x^3 + 3x - 1)\) is a polynomial of degree 5.

**Martin-Gay Interactive Videos**

Watch the section lecture video and answer the following questions.

5. From Examples 1–3, for what type of multiplication problem is the FOIL order of multiplication used?
6. Name at least one other method you can use to multiply Example 4.
7. From Example 5, why does multiplying the sum and difference of the same two terms always give you a binomial answer?
8. Why was the FOIL method not used for Example 10?

### 5.4 Exercise Set

Multiply using the FOIL method. See Examples 1 through 3.

1. \((x + 3)(x + 4)\)
2. \((x + 5)(x - 1)\)
3. \((x - 5)(x + 10)\)
4. \((y - 12)(y + 4)\)
5. \((5x - 6)(x + 2)\)
6. \((3y - 5)(2y - 7)\)
7. \((5y - 6)(4y - 1)\)
8. \((2x - 11)(2x - 9)\)
9. \((2x + 5)(3x - 1)\)
10. \((6x + 2)(x - 2)\)

11. \((x - \frac{1}{3})(x + \frac{2}{5})\)
12. \((x - \frac{2}{5})(x + \frac{1}{5})\)

Multiply. See Examples 4 and 5.

13. \((x + 2)^2\)
14. \((x + 7)^2\)
15. \((2x - 1)^2\)
16. \((7x - 3)^2\)
17. \((3a - 5)^2\)
18. \((5a + 2)^2\)
19. \((5x + 9)^2\)
20. \((6s - 2)^2\)
Multiply. See Example 6.

21. \((a - 7)(a + 7)\)
22. \((b + 3)(b - 3)\)
23. \((3x - 1)(3x + 1)\)
24. \((4x - 5)(4x + 5)\)
25. \(\left(3x - \frac{1}{2}\right)\left(3x + \frac{1}{2}\right)\)
26. \(\left(10x + \frac{2}{7}\right)\left(10x - \frac{2}{7}\right)\)
27. \((9x + y)(9x - y)\)
28. \((2x - y)(2x + y)\)
29. \((2x + 0.1)(2x - 0.1)\)
30. \((5x - 1.3)(5x + 1.3)\)

MIXED PRACTICE

Multiply. See Example 7.

31. \((a + 5)(a + 4)\)
32. \((a - 5)(a - 7)\)
33. \((a + 7)^2\)
34. \((b - 2)^2\)
35. \((4a + 1)(3a - 1)\)
36. \((6a + 7)(6a + 5)\)
37. \((x + 2)(x - 2)\)
38. \((x - 10)(x + 10)\)
39. \((3a + 1)^2\)
40. \((4a - 2)^2\)
41. \((x^2 + y)(4x - y^4)\)
42. \((x^3 - 2)(5x + y)\)
43. \((x + 3)(x^2 - 6x + 1)\)
44. \((x - 2)(x^2 - 4x + 2)\)
45. \((2a - 3)^2\)
46. \((5b - 4x)^2\)
47. \((5x - 6z)(5x + 6z)\)
48. \((11x - 7y)(11x + 7y)\)
49. \((x^5 - 3)(x^5 - 5)\)
50. \((a^4 + 5)(a^4 + 6)\)
51. \((x + 0.8)(x - 0.8)\)
52. \((y - 0.9)(y + 0.9)\)
53. \((a^3 + 11)(a^4 - 3)\)
54. \((x^5 + 5)(x^2 - 8)\)
55. \(3(x - 2)^2\)
56. \(2(3b + 7)^2\)
57. \((3b + 7)(2b - 5)\)
58. \((3y - 13)(y - 3)\)
59. \((7p - 8)(7p + 8)\)
60. \((3s - 4)(3s + 4)\)
61. \(\left(\frac{1}{3}a^2 - 7\right)\left(\frac{1}{3}a^2 + 7\right)\)

Express each as a product of polynomials in \(x\). Then multiply and simplify.

79. Find the area of the square rug shown if its side is \((2x + 1)\) feet.

80. Find the area of the rectangular canvas if its length is \((3x - 2)\) inches and its width is \((x - 4)\) inches.

REVIEW AND PREVIEW

Simplify each expression. See Section 5.1.

81. \(\frac{50b^{10}}{70b^5}\)
82. \(\frac{x^3y^6}{xy^2}\)
83. \(-\frac{8a^{17}b^{15}}{4a^7b^{10}}\)
84. \(-6a^8y\)
85. \(-\frac{2x^4y^{12}}{3x^4y^2}\)
86. \(-\frac{48ab^6}{32ab^3}\)
Find the slope of each line. See Section 3.4.

Find the area of the shaded figure. To do so, subtract the area of the smaller square(s) from the area of the larger geometric figure.

99. (5x - 3) meters
   (x + 1) m

100. (3x - 4) centimeters
     (3x + 4) centimeters

For Exercises 101 and 102, find the area of the shaded figure.

101. x 5
     y
     5

102. 2y 11
     y
     11

103. In your own words, describe the different methods that can be used to find the product (2x - 5)(3x + 1).
104. In your own words, describe the different methods that can be used to find the product (5x + 1)^2.
105. Suppose that a classmate asked you why (2x + 1)^2 is not (4x^2 + 1). Write down your response to this classmate.
106. Suppose that a classmate asked you why (2x + 1)^2 is not (4x^2 + 4x + 1). Write down your response to this classmate.
107. Using your own words, explain how to square a binomial such as (a + b)^2.
108. Explain how to find the product of two binomials using the FOIL method.

Find each product. For example,

\[
[(a + b) - 2][(a + b) + 2] = (a + b)^2 - 2^2 = a^2 + 2ab + b^2 - 4
\]

109. [(x + y) - 3][(x + y) + 3]
110. [(a + c) - 5][(a + c) + 5]
111. [(a - 3) + b][(a - 3) - b]
112. [(x - 2) + y][(x - 2) - y]
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CHAPTER 5  Exponents and Polynomials

Integrated Review  Exponents and Operations on Polynomials

Sections 5.1–5.4
Perform the indicated operations and simplify.

1. \((5x^2)(7x^3)\)
2. \((4y^2)(8y^7)\)
3. \(-4^2\)
4. \((-4)^2\)
5. \((x - 5)(2x + 1)\)
6. \((3x - 2)(x + 5)\)
7. \((x - 5) + (2x + 1)\)
8. \((3x - 2) + (x + 5)\)
9. \(\frac{7a^9y^{12}}{x^3y^{10}}\)
10. \(20a^2b^8\)
11. \(\frac{14a^2b^2}{3(4y - 3)(4y + 3)}\)
12. \((12m^7n^6)^2\)
13. \((3^{1/9})^3\)
14. \(2(7x - 1)(7x + 1)\)
15. \((x^7y^9)^0\)
16. \((10x^2 + 7x - 9) - (4x^2 - 6x + 2)\)
17. \((7x^2 - 2x + 3) - (5x^2 + 9)\)
18. \(7.88^2 - 6.8x + 3.3 + 0.6x^2 - 9\)
19. \(0.7y^2 - 1.2 + 1.8y^2 - 6y + 1\)
20. \((y - 9z)^2\)
21. \((x + 4y)^2\)
22. \((y - 9z)(y - 9z)\)
23. \((x + 4y)(x + 4y)\)
24. \((y - 9z)(y - 9z)\)
25. \(7x^2 - 6xy + 4(y^2 - xy)\)
26. \((x + 1)(x^2 - 3x - 2)\)
27. \((x - 3)(x^2 + 5x - 1)\)
28. \((5x^3 - 1)(4x^4 + 5)\)
29. \((2x^3 - 7)(3x^2 + 10)\)
30. \((5x^3 - 1)(x^2 + 2x - 3)\)
31. \((2x - 7)(x^2 - 6x + 1)\)
32. \((5x - 1)(x^2 + 2x - 3)\)

Perform the indicated operations and simplify if possible.

33. \(5x^3 + 5y^3\)
34. \((5x^3)(5y^3)\)
35. \((5x^3)^3\)
36. \(\frac{5x^3}{5y^3}\)
37. \(x + x\)
38. \(x \cdot x\)

5.5 Negative Exponents and Scientific Notation

OBJECTIVES

1. Simplify Expressions Containing Negative Exponents.
2. Use All the Rules and Definitions for Exponents to Simplify Exponential Expressions.
3. Write Numbers in Scientific Notation.
4. Convert Numbers from Scientific Notation to Standard Form.
5. Perform Operations on Numbers Written in Scientific Notation.

OBJECTIVE

1. Simplifying Expressions Containing Negative Exponents

Our work with exponential expressions so far has been limited to exponents that are positive integers or 0. Here we expand to give meaning to an expression like \(x^{-3}\).

Suppose that we wish to simplify the expression \(\frac{x^2}{x^5}\). If we use the quotient rule for exponents, we subtract exponents:

\[
\frac{x^2}{x^5} = x^{2-5} = x^{-3}, \quad x \neq 0
\]

But what does \(x^{-3}\) mean? Let’s simplify \(\frac{x^2}{x^3}\) using the definition of \(x^n\).

\[
\frac{x^2}{x^3} = \frac{x \cdot x}{x \cdot x \cdot x} = \frac{1}{x} \cdot \frac{x}{x}\]

Divide numerator and denominator by common factors by applying the fundamental principle for fractions.

\[
\frac{x^2}{x^3} = \frac{1}{x}
\]

If the quotient rule is to hold true for negative exponents, then \(x^{-3}\) must equal \(\frac{1}{x^3}\). From this example, we state the definition for negative exponents.

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Negative Exponents
If \( a \) is a real number other than 0 and \( n \) is an integer, then
\[
a^{-n} = \frac{1}{a^n}
\]

For example, \( x^{-3} = \frac{1}{x^3} \).

In other words, another way to write \( a^{-n} \) is to take its reciprocal and change the sign of its exponent.

**HELPFUL HINT**
Don’t forget that since there are no parentheses, only \( x \) is the base for the exponent \(-3\).

**EXAMPLE 1** Simplify by writing each expression with positive exponents only.

a. \( 3^{-2} \)  
   Solution:
   \[ 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \]  
   Use the definition of negative exponents.

b. \( 2x^{-3} \)  
   Solution:
   \[ 2x^{-3} = 2 \cdot \frac{1}{x^3} = \frac{2}{x^3} \]  
   Use the definition of negative exponents.

c. \( 2^{-1} + 4^{-1} \)  
   Solution:
   \[ 2^{-1} + 4^{-1} = \frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4} \]

d. \( (-2)^{-4} \)  
   Solution:
   \[ (-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{(-2)(-2)(-2)(-2)} = \frac{1}{16} \]

e. \( y^{-4} \)  
   Solution:
   \[ y^{-4} = \frac{1}{y^4} \]

**PRACTICE** Simplify by writing each expression with positive exponents only.

a. \( 5^{-3} \)  
   b. \( 3y^{-4} \)  
   c. \( 3^{-1} + 2^{-1} \)  
   d. \( (-5)^{-2} \)  
   e. \( x^{-5} \)

A negative exponent does not affect the sign of its base. Remember: Another way to write \( a^{-n} \) is to take its reciprocal and change the sign of its exponent: \( a^{-n} = \frac{1}{a^n} \). For example,

\[
\begin{align*}
  x^{-2} &= \frac{1}{x^2} \quad & 2^{-3} &= \frac{1}{2^3} \quad & \frac{1}{8} \\
  \frac{1}{y^{-4}} &= \frac{1}{y^4} \quad & \frac{1}{5^{-2}} &= \frac{1}{25} \quad & 25
\end{align*}
\]

From the preceding Helpful Hint, we know that \( x^{-2} = \frac{1}{x^2} \) and \( \frac{1}{y^{-4}} = y^4 \). We can use this to include another statement in our definition of negative exponents.

Negative Exponents
If \( a \) is a real number other than 0 and \( n \) is an integer, then
\[
a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n
\]

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### Simplifying Exponential Expressions

All the previously stated rules for exponents apply for negative exponents also. Here is a summary of the rules and definitions for exponents.

#### Summary of Exponent Rules

If \( m \) and \( n \) are integers and \( a, b, \) and \( c \) are real numbers, then:

- **Product rule for exponents:** \( a^m \cdot a^n = a^{m+n} \)
- **Power rule for exponents:** \((a^m)^n = a^{m \cdot n}\)
- **Power of a product:** \((ab)^n = a^n b^n\)
- **Power of a quotient:** \((\frac{a}{c})^n = \frac{a^n}{c^n}, \ c \neq 0\)
- **Quotient rule for exponents:** \(\frac{a^m}{a^n} = a^{m-n}, \ a \neq 0\)
- **Zero exponent:** \(a^0 = 1, \ a \neq 0\)
- **Negative exponent:** \(a^{-n} = \frac{1}{a^n}, \ a \neq 0\)
Example 4  Simplify the following expressions. Write each result using positive exponents only.

a. \((y^{-3}z^6)^{-6}\)  
\[ (y^{-3}z^6)^{-6} = y^{18}z^{-36} = \frac{y^{18}}{z^{36}} \]

b. \(\left(\frac{2x^3}{y^7}\right)^4\)  
\[ \left(\frac{2x^3}{y^7}\right)^4 = \frac{2^4 \cdot x^{12}}{y^{28}} = \frac{16 \cdot x^{12+1}}{y^{7+7}} = \frac{16x^{13}}{y^{14}} = 16x^6 \]
Use the power rule.

c. \(\left(\frac{3a^2}{b}\right)^{-3}\)  
\[ \left(\frac{3a^2}{b}\right)^{-3} = \frac{3^{-3} \cdot a^{-6}}{b^{-3}} = \frac{b^3}{3^3a^6} \]
Use the power rule.
\[ \frac{b^3}{27a^6} \]
Use the negative exponent rule.
\[ \frac{b^3}{27a^6} \]
Write 3³ as 27.

\[ \frac{4^{-1}x^{-3}y^{-6}}{4^{3}x^2y^{-6}} = 4^{-1-3}x^{-3-2}y^{-1-(-6)} = 4^2x^{-5}y^7 = \frac{4^2y^7}{x^5} = 16y^7 \]
\[ \frac{-2x^3y^3}{xy^{-1}} \]
\[ \frac{(-2)^3x^3y^3}{x^3} = -8x^6y^3 = -8x^9y^{-3}(-3) = -8x^6y^6 \]

Practice 4  Simplify the following expressions. Write each result using positive exponents only.

a. \((a^4b^{-3})^{-5}\)  
\[ (a^4b^{-3})^{-5} = a^{-20}b^{15} \]

b. \(\frac{x^2(x^5)^3}{x^7}\)  
\[ \frac{x^2(x^5)^3}{x^7} = \frac{x^2x^{15}}{x^7} = x^{17} \]

c. \(\left(\frac{5p^8}{q}\right)^{-2}\)  
\[ \left(\frac{5p^8}{q}\right)^{-2} = \left(\frac{q}{5p^8}\right)^{2} = \frac{q^2}{5^2p^{16}} = \frac{q^2}{25p^{16}} \]

Objective 3  Writing Numbers in Scientific Notation

Both very large and very small numbers frequently occur in many fields of science. For example, the distance between the Sun and the dwarf planet Pluto is approximately 5,906,000,000 kilometers, and the mass of a proton is approximately 0.00000000000000000000000165 gram. It can be tedious to write these numbers in this standard decimal notation, so scientific notation is used as a convenient shorthand for expressing very large and very small numbers.

Scientific Notation

A positive number is written in scientific notation if it is written as the product of a number \(a\), where \(1 \leq a < 10\), and an integer power \(r\) of 10:

\[ a \times 10^r \]

Mass of proton is approximately 0.000 000 000 000 000 000 000 000 001 65 gram
The numbers below are written in scientific notation. The \( \times \) sign for multiplication is used as part of the notation.

\[
2.03 \times 10^2 \quad 7.362 \times 10^7 \quad 5.906 \times 10^9 \\
1 \times 10^{-3} \quad 8.1 \times 10^{-5} \quad 1.65 \times 10^{-24}
\]

(Distance between the Sun and Pluto)  
(Mass of a proton)

The following steps are useful when writing numbers in scientific notation.

**To Write a Number in Scientific Notation**

**Step 1.** Move the decimal point in the original number to the left or right so that the new number has a value between 1 and 10 (including 1).

**Step 2.** Count the number of decimal places the decimal point is moved in Step 1. If the original number is 10 or greater, the count is positive. If the original number is less than 1, the count is negative.

**Step 3.** Multiply the new number in Step 1 by 10 raised to an exponent equal to the count found in Step 2.

**EXAMPLE 5** Write each number in scientific notation.

a. 367,000,000  
b. 0.000003  
c. 20,520,000,000  
d. 0.00085

**Solution**

a. **Step 1.** Move the decimal point until the number is between 1 and 10.

\[367,000,000\]

\[\text{8 places}\]

**Step 2.** The decimal point is moved 8 places, and the original number is 10 or greater, so the count is positive 8.

**Step 3.** \[367,000,000 = 3.67 \times 10^8\]

b. **Step 1.** Move the decimal point until the number is between 1 and 10.

\[0.000003\]

\[\text{6 places}\]

**Step 2.** The decimal point is moved 6 places, and the original number is less than 1, so the count is negative 6.

**Step 3.** \[0.000003 = 3.0 \times 10^{-6}\]

c. \[20,520,000,000 = 2.052 \times 10^{10}\]

d. \[0.00085 = 8.5 \times 10^{-4}\]

**PRACTICE 5** Write each number in scientific notation.

a. 0.000007  
b. 20,700,000  
c. 0.0043  
d. 812,000,000

**OBJECTIVE 4** Converting Numbers to Standard Form

A number written in scientific notation can be rewritten in standard form. For example, to write \(8.63 \times 10^3\) in standard form, recall that \(10^3 = 1000\).

\[8.63 \times 10^3 = 8.63(1000) = 8630\]

Notice that the exponent on the 10 is positive 3, and we moved the decimal point 3 places to the right.

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To write $7.29 \times 10^{-3}$ in standard form, recall that $10^{-3} = \frac{1}{1000}$. Therefore, $7.29 \times 10^{-3} = 7.29 \left( \frac{1}{1000} \right) = \frac{7.29}{1000} = 0.00729$.

The exponent on the 10 is negative 3, and we moved the decimal to the left 3 places.

In general, to write a scientific notation number in standard form, move the decimal point the same number of places as the exponent on 10. If the exponent is positive, move the decimal point to the right; if the exponent is negative, move the decimal point to the left.

**Example 6**

Write each number in standard notation, without exponents.

- a. $1.02 \times 10^5$
- b. $7.358 \times 10^{-3}$
- c. $8.4 \times 10^7$
- d. $3.007 \times 10^{-5}$

**Solution**

- a. Move the decimal point 5 places to the right.
  
  $1.02 \times 10^5 = 102,000$

- b. Move the decimal point 3 places to the left.
  
  $7.358 \times 10^{-3} = 0.007358$

- c. $8.4 \times 10^7 = 84,000,000$ 7 places to the right

- d. $3.007 \times 10^{-5} = 0.00003007$ 5 places to the left

**Practice 6**

Write each number in standard notation, without exponents.

- a. $3.67 \times 10^{-4}$
- b. $8.954 \times 10^6$
- c. $2.009 \times 10^{-5}$
- d. $4.054 \times 10^3$

**Concept Check**

Which number in each pair is larger?

- a. $7.8 \times 10^3$ or $2.1 \times 10^3$
- b. $9.2 \times 10^{-2}$ or $2.7 \times 10^4$
- c. $5.6 \times 10^{-4}$ or $6.3 \times 10^{-5}$

**Objective 5**

Performing Operations with Scientific Notation

Performing operations on numbers written in scientific notation uses the rules and definitions for exponents.

**Example 7**

Perform each indicated operation. Write each result in standard decimal notation.

- a. $(8 \times 10^{-6})(7 \times 10^3)$
- b. $\frac{12 \times 10^2}{6 \times 10^{-3}}$

**Solution**

- a. $(8 \times 10^{-6})(7 \times 10^3) = (8 \cdot 7) \times (10^{-6} \cdot 10^3)$
  
  $= 56 \times 10^{-3}$
  
  $= 0.056$

- b. $\frac{12 \times 10^2}{6 \times 10^{-3}} = \frac{12}{6} \times 10^{2-(3)} = 2 \times 10^5 = 200,000$

**Practice 7**

Perform each indicated operation. Write each result in standard decimal notation.

- a. $(5 \times 10^{-4})(8 \times 10^6)$
- b. $\frac{64 \times 10^3}{32 \times 10^{-7}}$

Answers to Concept Check:

- a. $2.1 \times 10^5$
- b. $2.7 \times 10^4$
- c. $5.6 \times 10^{-4}$
Scientific Notation
To enter a number written in scientific notation on a scientific calculator, locate the scientific notation key, which may be marked EE or EXP. To enter $3.1 \times 10^7$, press $3.1 \, \text{EE} \, 7$. The display should read $3.1 \, 07$.

Enter each number written in scientific notation on your calculator.
1. $5.31 \times 10^3$
2. $-4.8 \times 10^{14}$
3. $6.6 \times 10^{-9}$
4. $-9.9811 \times 10^{-2}$

Multiply each of the following on your calculator. Notice the form of the result.
5. $3,000,000 \times 5,000,000$
6. $230,000 \times 1000$

Multiply each of the following on your calculator. Write the product in scientific notation.
7. $1 \times 3.26 \times 10^6\times 21$
8. $21 \times 1.237 \times 10^9$

Vocabulary, Readiness & Video Check
Fill in each blank with the correct choice.

1. The expression $x^{-3}$ equals ______.
   a. $-x^3$ b. $\frac{1}{x^3}$ c. $\frac{-1}{x^3}$ d. $\frac{1}{x^3}$
2. The expression $5^{-4}$ equals ______.
   a. $-20$ b. $-625$ c. $\frac{1}{20}$ d. $\frac{1}{625}$
3. The number $3.021 \times 10^{-3}$ is written in ______.
   a. standard form b. expanded form c. scientific notation
4. The number $0.0261$ is written in ______.
   a. standard form b. expanded form c. scientific notation

Watch the section lecture video and answer the following questions.

5. What important reminder is made at the end of Example 1?
6. Name all the rules and definitions used to simplify Example 8.
7. From Examples 9 and 10, explain how the movement of the decimal point in step 1 suggests the sign of the exponent on the number 10.
8. From Example 11, what part of a number written in scientific notation is key in telling you how to write the number in standard form?
9. For Example 13, what exponent rules were needed to evaluate?

5.5 Exercise Set
Simplify each expression. Write each result using positive exponents only. See Examples 1 through 3.

1. $4^{-3}$
2. $6^{-2}$
3. $(3^{-4})^{-3}$
4. $(-3)^{-5}$
5. $7x^{-3}$
6. $(7x)^{-3}$
7. $(\frac{1}{7})^{-5}$
8. $(\frac{1}{8})^{-2}$
9. $(-\frac{1}{4})^{-3}$
10. $(\frac{1}{8})^{-2}$
11. $3^{-1} + 5^{-1}$
12. $4^{-1} + 4^{-2}$
MIXED PRACTICE

Simplify each expression. Write each result using positive exponents only. See Examples 1 through 4.

13. \( \frac{1}{p^{-3}} \) 14. \( \frac{1}{q^{-5}} \) 15. \( \frac{p^{-5}}{q} \)  
16. \( \frac{r^{-5}}{s^{-7}} \) 17. \( \frac{x^{-2}}{x} \) 18. \( \frac{y}{y^3} \)  
19. \( \frac{z^{-4}}{z^{-7}} \) 20. \( \frac{x^{-4}}{x^{-1}} \) 21. \( 3^{-2} + 3^{-1} \)  
22. \( 4^{-2} - 4^{-3} \) 23. \( -\frac{1}{p^{-4}} \) 24. \( -\frac{1}{y^6} \)  
25. \( -2^0 - 3^0 \) 26. \( 5^0 + (-5)^0 \)  

Write each number in scientific notation. See Example 5.

69. 78,000  
70. 9,300,000,000  
71. 0.00000167  
72. 0.00000017  
73. 0.00635  
74. 0.00194  
75. 1,160,000  
76. 700,000  

77. More than 2,000,000,000 pencils are manufactured in the United States annually. Write this number in scientific notation. (Source: AbsoluteTrivia.com)  
78. The temperature at the interior of the Earth is 20,000,000 degrees Celsius. Write 20,000,000 in scientific notation.  
79. As of this writing, the world’s largest optical telescope is the Gran Telescopio Canarias, located in La Palma, Canary Islands, Spain. The elevation of this telescope is 2400 meters above sea level. Write 2400 in scientific notation.

80. In March 2004, the European Space Agency launched the Rosetta spacecraft, whose mission was to deliver the Philae lander to explore comet 67P/Churyumov-Gerasimenko. The lander finally arrived on the comet in late 2014. This comet is currently more than 320,000,000 miles from Earth. Write 320,000,000 in scientific notation. (Source: European Space Agency)

Write each number in standard notation. See Example 6.

81. \( 8.673 \times 10^{-10} \) 82. \( 9.056 \times 10^{-4} \) 83. \( 3.3 \times 10^{-2} \)
84. \(4.8 \times 10^{-6}\)
85. \(2.032 \times 10^4\)
86. \(9.07 \times 10^{10}\)
87. Each second, the Sun converts \(7.0 \times 10^8\) tons of hydrogen into helium and energy in the form of gamma rays. Write this number in standard notation. (Source: Students for the Exploration and Development of Space)
88. In chemistry, Avogadro’s number is the number of atoms in one mole of an element. Avogadro’s number is \(6.02214199 \times 10^{23}\). Write this number in standard notation. (Source: National Institute of Standards and Technology)
89. The distance light travels in 1 year is \(9.46 \times 10^{12}\) kilometers. Write this number in standard notation.
90. The population of the world is \(7.3 \times 10^9\). Write this number in standard notation. (Source: UN World Population Clock)

**MIXED PRACTICE**

See Examples 5 and 6. Below are some interesting facts about selected countries’ external debts at a certain time. These are public and private debts owed to nonresidents of that country. If a number is written in standard form, write it in scientific notation. If a number is written in scientific notation, write it in standard form. (Source: CIA World Factbook)

<table>
<thead>
<tr>
<th>Selected Countries and Their External Debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
</tr>
<tr>
<td>Russia</td>
</tr>
<tr>
<td>Mexico</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td>Indonesia</td>
</tr>
</tbody>
</table>

91. The external debt of Russia at a certain time was $714,000,000,000.
92. The amount by which Russia’s debt was greater than Mexico’s debt was $359,000,000,000.
93. At a certain time, China’s external debt was $8.63 \times 10^{11}.
94. At a certain time, the external debt of the United States was $1.5 \times 10^{13}.
95. At a certain time, the estimated per person share of the United States external debt was $4.7 \times 10^4.
96. The bar graph shows the external debt of five countries. Estimate the height of the tallest bar and the shortest bar in standard notation. Then write each number in scientific notation.

Evaluate each expression using exponential rules. Write each result in standard notation. See Example 7
97. \((1.2 \times 10^{-3})(3 \times 10^{-2})\)
98. \((2.5 \times 10^5)(2 \times 10^{-6})\)

99. \((4 \times 10^{-10})(7 \times 10^{-9})\)
100. \((5 \times 10^6)(4 \times 10^{-8})\)
101. \(8 \times 10^{-1}\) \(16 \times 10^8\)
102. \(25 \times 10^{-4}\) \(5 \times 10^{-9}\)
103. \(1.4 \times 10^{-2}\) \(7 \times 10^{-8}\)
104. \(0.4 \times 10^2\) \(0.2 \times 10^{11}\)

**REVIEW AND PREVIEW**

Simplify the following. See Section 5.1.
105. \(\frac{5x^7}{3x^3}\)
106. \(\frac{27y^{14}}{3y}\)
107. \(\frac{15z^4y^3}{21z^y}\)
108. \(\frac{18a^7b^{17}}{30a^b}\)

Use the distributive property and multiply. See Sections 5.3 and 5.5.
109. \(\frac{1}{2}(5y^2 - 6y + 5)\)
110. \(\frac{2}{5}(3x^5 + x^4 - 2)\)

**CONCEPT EXTENSIONS**

\(\triangle 111\). Find the volume of the cube.

\(\triangle 112\). Find the area of the triangle.

\(\triangle 113\). \((2a^3)^4a^4 + a^5a^8\)
\(\triangle 114\). \((2a^3)^3a^{-3} + a^{11}a^{-5}\)

Fill in the boxes so that each statement is true. (More than one answer may be possible for these exercises.)

115. \(x^\Box = \frac{1}{x^3}\)
116. \(7^\Box = \frac{1}{49}\)
117. \(z^{\Box}z^{\Box} = z^{-10}\)
118. \((x^\Box)^\Box = x^{-15}\)
119. Which is larger? See the Concept Check in this section.
   a. \(9.7 \times 10^{-2}\) or \(1.3 \times 10^1\)
   b. \(8.6 \times 10^2\) or \(4.4 \times 10^7\)
   c. \(6.1 \times 10^{-2}\) or \(5.6 \times 10^{-4}\)
120. Determine whether each statement is true or false.
   a. \(5^{-1} < 5^{-2}\)
   b. \(\left(\frac{1}{5}\right)^{-1} < \left(\frac{1}{5}\right)^{-2}\)
   c. \(a^{-1} < a^{-2}\) for all nonzero numbers.
Section 5.6  Dividing Polynomials

5.6 Dividing Polynomials

OBJECTIVES

1 Divide a Polynomial by a Monomial.
2 Use Long Division to Divide a Polynomial by Another Polynomial.

OBJECTIVE 1  Dividing by a Monomial

Now that we know how to add, subtract, and multiply polynomials, we practice dividing polynomials.

To divide a polynomial by a monomial, recall addition of fractions. Fractions that have a common denominator are added by adding the numerators:

\[
\frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}
\]

If we read this equation from right to left and let \(a, b,\) and \(c\) be monomials, \(c \neq 0,\) we have the following:

**Dividing a Polynomial by a Monomial**

Divide each term of the polynomial by the monomial.

\[
\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}, \quad c \neq 0
\]

Throughout this section, we assume that denominators are not 0.

**EXAMPLE 1**  Divide \(6m^2 + 2m\) by \(2m.\)

**Solution** We begin by writing the quotient in fraction form. Then we divide each term of the polynomial \(6m^2 + 2m\) by the monomial \(2m.\)

\[
\frac{6m^2 + 2m}{2m} = \frac{6m^2}{2m} + \frac{2m}{2m} = 3m + 1 \quad \text{Simplify.}
\]

**Check:** We know that if \(\frac{6m^2 + 2m}{2m} = 3m + 1,\) then \(2m \cdot (3m + 1)\) must equal \(6m^2 + 2m.\) Thus, to check, we multiply.

\[
2m(3m + 1) = 2m(3) + 2m(1) = 6m^2 + 2m
\]

The quotient \(3m + 1\) checks.

**PRACTICE**  Divide \(8t^3 + 4t^2\) by \(4t^2.\)
EXAMPLE 2
Divide: \( \frac{9x^5 - 12x^2 + 3x}{3x^2} \)

Solution
\[
\frac{9x^5 - 12x^2 + 3x}{3x^2} = \frac{9x^5}{3x^2} - \frac{12x^2}{3x^2} + \frac{3x}{3x^2}
\]
\[
= 3x^3 - 4 + \frac{1}{x}
\]

Notice that the quotient is not a polynomial because of the term \( \frac{1}{x} \). This expression is called a rational expression—we will study rational expressions further in Chapter 7. Although the quotient of two polynomials is not always a polynomial, we may still check by multiplying.

Check:
\[
3x^2 \left( 3x^3 - 4 + \frac{1}{x} \right) = 3x^2 \left( 3x^3 \right) - 3x^2 \left( 4 \right) + 3x^2 \left( \frac{1}{x} \right)
\]
\[
= 9x^5 - 12x^2 + 3x
\]

EXAMPLE 3
Divide: \( \frac{8x^2y^2 - 16xy + 2x}{4xy} \)

Solution
\[
\frac{8x^2y^2 - 16xy + 2x}{4xy} = \frac{8x^2y^2}{4xy} - \frac{16xy}{4xy} + \frac{2x}{4xy}
\]
\[
= 2xy - 4 + \frac{1}{2y}
\]

Check:
\[
4xy \left( 2xy - 4 + \frac{1}{2y} \right) = 4xy \left( 2xy \right) - 4xy \left( 4 \right) + 4xy \left( \frac{1}{2y} \right)
\]
\[
= 8x^2y^2 - 16xy + 2x
\]

CONCEPT CHECK
In which of the following is \( \frac{x + 5}{5} \) simplified correctly?

a. \( \frac{x}{5} + 1 \)  
b. \( x \)  
c. \( x + 1 \)

OBJECTIVE 2 Using Long Division to Divide by a Polynomial

To divide a polynomial by a polynomial other than a monomial, we use a process known as long division. Polynomial long division is similar to number long division, so we review long division by dividing 13 into 3660.

Answer to Concept Check:

a
The quotient is 281R7, which can be written as $281 \frac{7}{13}$ remainder divisor

Recall that division can be checked by multiplication. To check a division problem such as this one, we see that

$$13 \cdot 281 + 7 = 3660$$

Now we demonstrate long division of polynomials.

**EXAMPLE 4** Divide $x^2 + 7x + 12$ by $x + 3$ using long division.

**Solution**

To subtract, change the signs of these terms and add.

\[
\begin{array}{c|ccr}
\hline
x & + & 3 & x & + & 7 & + & 12 \\
\hline
x^2 & + & 3x^2 & + & 7x & + & 12 \\
\hline
x^2 & + & 3x & \hline
4x & + & 12 \\
-4x & \hline
0 & \\
\hline
\end{array}
\]

How many times does $x$ divide $x^2 + 7x + 12$? $x(x + 3)$

Subtract and bring down the next term.

The quotient is $x + 4$.

**Check:** We check by multiplying.

\[
\begin{array}{c|crr}
\hline
\text{divisor} & \cdot & \text{quotient} & + & \text{remainder} & = & \text{dividend} \\
\hline
(x + 3) & \cdot & (x + 4) & + & 0 & = & x^2 + 7x + 12 \\
\hline
\end{array}
\]

The quotient checks.

**PRACTICE 4** Divide $x^2 + 5x + 6$ by $x + 2$ using long division.

**EXAMPLE 5** Divide $6x^2 + 10x - 5$ by $3x - 1$ using long division.

**Solution**

\[
\begin{array}{c|ccr}
\hline
2x & + & 4 \\
\hline
3x & - & 1 & 6x^2 & + & 10x & - & 5 \\
\hline
3x & = & 2x & \text{is a term of the quotient.} \\
6x^2 & + & 2x & \hline
12x & - & 5 \\
\hline
12x & + & 4 \\
\hline
-1 & \hline
\end{array}
\]

Multiply $2x(3x - 1)$

Subtract and bring down the next term.

Multiply $4(3x - 1)$

Subtract. The remainder is $-1$.

Thus $(6x^2 + 10x - 5)$ divided by $(3x - 1)$ is $(2x + 4)$ with a remainder of $-1$. This can be written as

$$\frac{6x^2 + 10x - 5}{3x - 1} = 2x + 4 + \frac{-1}{3x - 1} \quad \text{remainder divisor}$$

**Check:** To check, we multiply $(3x - 1)(2x + 4)$ Then we add the remainder, $-1$, to this product.

\[
(3x - 1)(2x + 4) + (-1) = (6x^2 + 12x - 2x - 4) - 1 = 6x^2 + 10x - 5
\]

The quotient checks.

**PRACTICE 5** Divide $4x^2 + 8x - 7$ by $2x + 1$ using long division.
In Example 5, the degree of the divisor, \(3x - 1\), is 1 and the degree of the remainder, \(-1\), is 0. The division process is continued until the degree of the remainder polynomial is less than the degree of the divisor polynomial.

Writing the dividend and divisor in a form with descending order of powers and with no missing terms is helpful when dividing polynomials.

**Example 6**
Divide: \(\frac{4x^2 + 7 + 8x^3}{2x + 3}\)

**Solution** Before we begin the division process, we rewrite
\[
4x^2 + 7 + 8x^3 \quad \text{as} \quad 8x^3 + 4x^2 + 0x + 7
\]
Notice that we have written the polynomial in descending order and have represented the missing \(x^1\)-term by 0x.

\[
\begin{array}{c|ccccc}
& 4x^2 & -4x & +6 \\
\hline
2x & 3x^3 & +4x^2 & +0x & +7 \\
\hline
& -8x^2 & +0x \\
\hline
& +8x^2 & +12x \\
\hline
& 12x & +7 \\
\hline
& -12x & -18 \\
\hline
& -11 & \text{Remainder}
\end{array}
\]

Thus, \(\frac{4x^2 + 7 + 8x^3}{2x + 3} = 4x^3 - 4x + 6 + \frac{-11}{2x + 3}\). 

**Practice 6**
Divide: \(\frac{11x - 3 + 9x^3}{3x + 2}\)

**Example 7**
Divide: \(\frac{2x^4 - x^3 + 3x^2 + x - 1}{x^2 + 1}\)

**Solution** Before dividing, rewrite the divisor polynomial
\(x^2 + 1\) as \(x^2 + 0x + 1\)
The 0x term represents the missing \(x^1\)-term in the divisor.

\[
\begin{array}{c|cccc}
& 2x^2 & -x & +1 \\
\hline
x^2 & +0x & +1 \\
\hline
& x^2 & +x \\
\hline
& +x^2 & +x \\
\hline
& x^2 & +2x & -1 \\
\hline
& -x^2 & -0x & +1 \\
\hline
& 2x & -2 & \text{Remainder}
\end{array}
\]

Thus, \(\frac{2x^4 - x^3 + 3x^2 + x - 1}{x^2 + 1} = 2x^2 - x + 1 + \frac{2x - 2}{x^2 + 1}\). 

**Practice 7**
Divide: \(\frac{3x^4 - 2x^3 - 3x^2 + x + 4}{x^2 + 2}\)
EXAMPLE 8 Divide \( x^3 - 8 \) by \( x - 2 \).

Solution: Notice that the polynomial \( x^3 - 8 \) is missing an \( x^2 \)-term and an \( x \)-term. We'll represent these terms by inserting \( 0x^2 \) and \( 0x \).

\[
\begin{align*}
x^2 + 2x + 4 \\
x - 2 \quad x^3 + 0x^2 + 0x - 8 \\
\frac{x^3 - 8}{x^2} \\
\frac{2x^2}{2x} + 0x \\
\frac{2x^2}{4x} \quad 4x - 8 \\
\frac{-4x}{-4} \quad 8 \\
0
\end{align*}
\]

Thus, \( \frac{x^3 - 8}{x - 2} = x^2 + 2x + 4 \).

Check: To check, see that \( (x^2 + 2x + 4)(x - 2) = x^3 - 8 \).

Practice 8 Divide \( x^3 + 27 \) by \( x + 3 \).

Vocabulary, Readiness & Video Check

Use the choices below to fill in each blank. Choices may be used more than once.

- dividend
- divisor
- quotient

1. In \( \frac{3}{6} \), the 18 is the _____, the 3 is the _____, and the 6 is the _____.

2. In \( \frac{x + 2}{x + 1} \), the \( x + 1 \) is the _____, the \( x^2 + 3x + 2 \) is the _____, and the \( x + 2 \) is the _____.

Simplify each expression mentally.

3. \( \frac{a^6}{a^2} \)  
4. \( \frac{p^8}{p^3} \)  
5. \( \frac{y^2}{y} \)  
6. \( \frac{a^3}{a} \)

Martin-Gay Interactive Videos

Watch the section lecture video and answer the following questions.

7. The lecture before Example 1 begins with adding two fractions with the same denominator. From there, the lecture continues to a method for dividing a polynomial by a monomial. What role does the monomial play in the fraction example?

8. In Example 5, you're told that although you don't have to fill in missing powers in the divisor and the dividend, it really is a good idea to do so. Why?

5.6 Exercise Set

Perform each division. See Examples 1 through 3.

1. \( \frac{12x^4 + 3x^2}{x} \)  
2. \( \frac{15x^2 - 9x^3}{x} \)  
3. \( \frac{20x^3 - 30x^2 + 5x + 5}{5} \)  
4. \( \frac{8x^3 - 4x^2 + 6x + 2}{2} \)
Find each quotient using long division. See Examples 4 and 5.

13. \(\frac{x^2 + 4x + 3}{x + 3}\)
14. \(\frac{x^2 + 7x + 10}{x + 5}\)
15. \(\frac{2x^2 + 13x + 15}{x + 5}\)
16. \(\frac{3x^2 + 8x + 4}{x + 2}\)
17. \(\frac{2x^2 - 7x + 3}{x - 4}\)
18. \(\frac{3x^2 - x - 4}{x - 1}\)
19. \(\frac{9x^3 - 3a^2 - 3a + 4}{3a + 2}\)
20. \(\frac{4x^3 + 12x^2 + x - 14}{2x + 3}\)
21. \(\frac{8x^2 + 10x + 1}{2x + 1}\)
22. \(\frac{3x^2 + 17x + 7}{3x + 2}\)
23. \(\frac{2x^3 + 2x^2 - 17x + 8}{x - 2}\)
24. \(\frac{4x^3 + 11x^2 - 8x - 10}{x + 3}\)

Find each quotient using long division. Don’t forget to write the polynomials in descending order and fill in any missing terms. See Examples 6 through 8.

25. \(\frac{x^2 - 36}{x - 6}\)
26. \(\frac{a^2 - 49}{a - 7}\)
27. \(\frac{x^3 - 27}{x - 3}\)
28. \(\frac{x^3 + 64}{x + 4}\)
29. \(\frac{1 - 3x^2}{x + 2}\)
30. \(\frac{7 - 5x^2}{x + 3}\)
31. \(\frac{-4b + 4b^2 - 5}{2b - 1}\)
32. \(\frac{-3y + 2y^2 - 15}{2y + 5}\)

MIXED PRACTICE

Divide. If the divisor contains 2 or more terms, use long division.

33. \(\frac{a^2b^2 - ab^3}{ab}\)
34. \(\frac{m^3n^2 - mn^4}{mn}\)
35. \(\frac{8x^2 + 6x - 27}{2x - 3}\)
36. \(\frac{18w^2 + 18w - 8}{3w + 4}\)
37. \(\frac{2x^2y + 8x^2y^2 - xy^2}{2xy}\)
38. \(\frac{11x^3y^3 - 33xy + x^2y^3}{11xy}\)
39. \(\frac{2b^3 + 9b^2 + 6b - 4}{b + 4}\)
40. \(\frac{2x^3 + 3x^2 - 3x + 4}{x + 2}\)
41. \(\frac{5x^2 + 28x - 10}{x + 6}\)
42. \(\frac{2x^2 + x - 15}{x + 3}\)
43. \(\frac{10x^3 - 24x^2 - 10x}{10x}\)
44. \(\frac{2x^3 + 12x^2 + 16}{4x^2}\)
45. \(\frac{6x^2 + 17x - 4}{x + 3}\)
46. \(\frac{2x^2 - 9x + 15}{x - 6}\)
47. \(\frac{30x^2 - 17x + 2}{5x - 2}\)
48. \(\frac{4x^2 - 13x - 12}{4x + 3}\)
49. \(\frac{3x^4 - 9x^3 + 12}{-3x}\)
50. \(\frac{8y^6 - 3y^2 - 4y}{4y}\)
51. \(\frac{x^3 + 6x^2 + 18x + 27}{x + 3}\)
52. \(\frac{x^3 - 8x^2 + 32x - 64}{x - 4}\)
53. \(\frac{y^3 + 3y^2 + 4}{y - 2}\)
Section 5.6 Dividing Polynomials

54. \[
\frac{3x^3 + 11x + 12}{x + 4}
\]
55. \[
\frac{5 - 6x^2}{x - 2}
\]
56. \[
\frac{3 - 7x^2}{x - 3}
\]

Divide.
57. \[
\frac{x^2 + x}{x^2 + x}
\]
58. \[
\frac{x - x^4}{x^3 + 1}
\]

REVIEW AND PREVIEW
Multiply each expression. See Section 5.3.
59. \(2a(a^2 + 1)\)
60. \(-4a(3a^2 - 4)\)
61. \(2x(x^2 + 7x - 5)\)
62. \(4y(y^2 - 8y - 4)\)
63. \(-3xy(xy^2 + 7x^2y + 8)\)
64. \(-9xy(4xyz + 7x^2y^2 + 2)\)
65. \(9ab(ab^2c + 4bc - 8)\)
66. \(-7sr(6s^3r + 9sr^3 + 9rs + 8)\)

CONCEPT EXTENSIONS
67. The perimeter of a square is \((12x^3 + 4x - 16)\) feet. Find the length of its side.

68. The volume of the swimming pool shown is \((36x^3 - 12x^2 + 6x^2)\) cubic feet. If its height is \(2x\) feet and its width is \(3x\) feet, find its length.

69. In which of the following is \(\frac{a + 7}{7}\) simplified correctly? See the Concept Check in this section.
   a. \(a + 1\)  
   b. \(a\)  
   c. \(\frac{a}{7} + 1\)

70. In which of the following is \(\frac{5x + 15}{5}\) simplified correctly? See the Concept Check in this section.
   a. \(x + 15\)  
   b. \(x + 3\)  
   c. \(x + 1\)

71. Explain how to check a polynomial long division result when the remainder is 0.

72. Explain how to check a polynomial long division result when the remainder is not 0.

73. The area of the following parallelogram is \((10x^2 + 31x + 15)\) square meters. If its base is \((5x + 3)\) meters, find its height.

74. The area of the top of the Ping-Pong table is \((49x^2 + 70x - 200)\) square inches. If its length is \((7x + 20)\) inches, find its width.

75. \((18x^{10a} - 12x^{8a} + 14x^{5a} - 2x^{3a}) + 2x^{3a}\)
76. \((25y^{11b} + 5y^{6b} - 20y^{3b} + 100y^{b}) + 5y^{b}\)


Chapter 5 Vocabulary Check

Fill in each blank with one of the words or phrases listed below.

<table>
<thead>
<tr>
<th>term</th>
<th>coefficient</th>
<th>monomial</th>
<th>binomial</th>
<th>trinomial</th>
<th>polynomials</th>
<th>degree of a term</th>
<th>distributive</th>
<th>FOIL</th>
<th>degree of a polynomial</th>
</tr>
</thead>
</table>

1. A _______________ is a number or the product of numbers and variables raised to powers.
2. The _______________ method may be used when multiplying two binomials.
3. A polynomial with exactly three terms is called a _______________.
4. The _______________ is the greatest degree of any term of the polynomial.
5. A polynomial with exactly two terms is called a _______________.
6. The _______________ of a term is its numerical factor.
7. The _______________ is the sum of the exponents on the variables in the term.
8. A polynomial with exactly one term is called a _______________.
9. Monomials, binomials, and trinomials are all examples of _______________.
10. The _______________ property is used to multiply 2x(x - 4).

Chapter 5 Highlights

<table>
<thead>
<tr>
<th>DEFINITIONS AND CONCEPTS</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Section 5.1 Exponents</strong></td>
<td></td>
</tr>
</tbody>
</table>

\[ a^n \text{ means the product of } n \text{ factors, each of which is } a. \]

If \( m \) and \( n \) are integers and no denominators are 0,

- **Product Rule:** \( a^m \cdot a^n = a^{m+n} \)
- **Power Rule:** \( (a^m)^n = a^{mn} \)
- **Power of a Product Rule:** \( (ab)^n = a^nb^n \)
- **Power of a Quotient Rule:** \( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \)
- **Quotient Rule:** \( \frac{a^m}{a^n} = a^{m-n} \)
- **Zero Exponent:** \( a^0 = 1, \ a \neq 0. \)

- \[ 3^2 = 3 \cdot 3 = 9 \]
- \[ (-5)^3 = (-5)(-5)(-5) = -125 \]
- \[ \left( \frac{1}{2} \right)^4 = \frac{1^4}{2^4} = \frac{1}{16} \]
- \[ x^2 \cdot x^7 = x^{2+7} = x^9 \]
- \[ (5^3)^8 = 5^{24} \]
- \[ (7y)^4 = 7^4y^4 \]
- \[ \left( \frac{x}{y} \right)^3 = \frac{x^3}{y^3} \]
- \[ \frac{x^6}{x^4} = x^{6-4} = x^2 \]
- \[ 5^0 = 1, x^0 = 1, x \neq 0 \]
Chapter 5 Highlights

DEFINITIONS AND CONCEPTS

Section 5.2 Adding and Subtracting Polynomials

A term is a number or the product of numbers and variables raised to powers.

The numerical coefficient or coefficient of a term is its numerical factor.

A polynomial is a term or a finite sum of terms in which variables may appear in the numerator raised to whole number powers only.

A monomial is a polynomial with exactly 1 term.
A binomial is a polynomial with exactly 2 terms.
A trinomial is a polynomial with exactly 3 terms.

The degree of a term is the sum of the exponents on the variables in the term.

The degree of a polynomial is the greatest degree of any term of the polynomial.

To add polynomials, add or combine like terms.

To subtract two polynomials, change the signs of the terms of the second polynomial, then add.

Section 5.3 Multiplying Polynomials

To multiply two polynomials, multiply each term of one polynomial by each term of the other polynomial and then combine like terms.

Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7x^2$</td>
<td>7</td>
</tr>
<tr>
<td>$y$</td>
<td>1</td>
</tr>
<tr>
<td>$-a^2b$</td>
<td>-1</td>
</tr>
</tbody>
</table>

Polynomials

<table>
<thead>
<tr>
<th>Polynomials</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x^2 - 2x + 1$</td>
<td>2 (Trinomial)</td>
</tr>
<tr>
<td>$-0.2a^2b - 5b^2$</td>
<td>2 (Binomial)</td>
</tr>
<tr>
<td>$\frac{5}{6}y^3$</td>
<td>3 (Monomial)</td>
</tr>
</tbody>
</table>

Term Degree

<table>
<thead>
<tr>
<th>Term</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-5x^3$</td>
<td>3</td>
</tr>
<tr>
<td>3 (or $3x^0$)</td>
<td>0</td>
</tr>
<tr>
<td>$2a^2b^2c$</td>
<td>5</td>
</tr>
</tbody>
</table>

Polynomial Degree

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5x^2 - 3x + 2$</td>
<td>2</td>
</tr>
<tr>
<td>$7y + 8y^2 - 12$</td>
<td>2 + 3 = 5</td>
</tr>
</tbody>
</table>

Add:

$(7x^2 - 3x + 2) + (-5x - 6) = 7x^2 - 3x + 2 - 5x - 6 = 7x^2 - 8x - 4$

Subtract:

$(17y^2 - 2y + 1) - (-3y^3 + 5y - 6) = (17y^2 - 2y + 1) + (3y^3 - 5y + 6) = 17y^2 - 2y + 1 + 3y^3 - 5y + 6 = 3y^3 + 17y^2 - 7y + 7$

Multiply:

$(2x + 1)(5x^2 - 6x + 2) = 2x(5x^2 - 6x + 2) + 1(5x^2 - 6x + 2) = 10x^3 - 12x^2 + 4x + 5x^2 - 6x + 2 = 10x^3 - 7x^2 - 2x + 2$
### Definitions and Concepts

#### Section 5.4 Special Products

**The FOIL method** may be used when multiplying two binomials.

#### Squaring a Binomial

\[(a + b)^2 = a^2 + 2ab + b^2\]

\[(a - b)^2 = a^2 - 2ab + b^2\]

#### Multiplying the Sum and Difference of Two Terms

\[(a + b)(a - b) = a^2 - b^2\]

**Examples**

Multiply: \((5x - 3)(2x + 3)\)

\[
\begin{array}{c|c|c|c|c}
    & F & O & I & L \\
    \hline
    5x & 10x^2 & 15x & -15x & -9x \\
    -3 & -6x & -9 & +9x & +27 \\
    \hline
    & 10x^2 & 15x & -9x & -9 \\
\end{array}
\]

Square each binomial.

\((x + 5)^2 = x^2 + 10x + 25\)

\((3x - 2y)^2 = 9x^2 - 12xy + 4y^2\)

Multiply:

\((6y + 5)(6y - 5) = (6y)^2 - 5^2\)

\[= 36y^2 - 25\]

### Section 5.5 Negative Exponents and Scientific Notation

If \(a \neq 0\) and \(n\) is an integer,

\[a^{-n} = \frac{1}{a^n}\]

Rules for exponents are true for positive and negative integers.

A positive number is written in scientific notation if it is written as the product of a number \(a\), \(1 \leq a < 10\), and an integer power \(r\) of 10.

\[a \times 10^r\]

**Examples**

\[3^{-2} = \frac{1}{3^2} = \frac{1}{9} \times 3^{-2} = \frac{5}{x^2}\]

Simplify: \(\left(\frac{x^7y}{x^3}\right)^{-2}\)

\[= \frac{x^{14}y^{-2}}{x^{-10}}\]

\[= x^{4(-10)}y^{-2}\]

\[= x^{14}y^{-2}\]

Write each number in scientific notation.

\[12,000 = 1.2 \times 10^4\]

\[0.00000568 = 5.68 \times 10^{-6}\]

### Section 5.6 Dividing Polynomials

To divide a polynomial by a monomial:

\[
\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}
\]

To divide a polynomial by a polynomial other than a monomial, use long division.

Divide:

\[
\frac{15x^5 - 10x^3 + 5x^2 - 2x}{5x^3 - 10x^3 + 5x^2 - 2x} = \frac{15x^5}{5x^3} - \frac{10x^3}{5x^3} + \frac{5x^2}{5x^2} - \frac{2x}{5x^2}
\]

\[= 3x^2 - 2x + 1 - \frac{2}{5x}\]

\[= 5x - 1 + \frac{-4}{2x + 3}\]

\[2x + 3\frac{10x^2 + 13x - 7}{10x^2 + 15x} - \frac{2x - 7}{2x + 3} - \frac{4}{3} - 4\]

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Chapter 5 Review

(5.1) State the base and the exponent for each expression.
1. $7^9$
2. $(-5)^4$
3. $-5^4$
4. $x^6$

Evaluate each expression.
5. $8^3$
6. $(-6)^2$
7. $-6^2$
8. $-4^2 - 4^0$
9. $(3b)^0$
10. $8b$

Simplify each expression.
11. $y^2 \cdot y^7$
12. $x^9 \cdot x^3$
13. $(2x^3)(-3x^6)$
14. $(-5y^3)(4y^4)$
15. $(x^4)^2$
16. $(y^3)^5$
17. $(3b)^4$
18. $(2x^3)^3$
19. $\frac{x^9}{x^4}$
20. $\frac{z^{12}}{z^5}$
21. $\frac{a^3b^4}{ab}$
22. $\frac{x^{10}}{y^6}$
23. $\frac{3x^4y^{10}}{12xy^6}$
24. $\frac{2x^7y}{8xy^2}$
25. $5a^2(2a^3)^3$
26. $(2x)^2(9x)$
27. $(-5a^0) + 7^0 + 8^0$
28. $8x^0 + 9^0$

Simplify the given expression and choose the correct result.
29. \(\frac{3x^4}{4y}\)
   a. \(\frac{7x^{64}}{64y^3}\)
   b. \(\frac{27x^{12}}{64y^3}\)
   c. \(\frac{9x^{12}}{12y^3}\)
   d. \(\frac{3x^{12}}{4y^3}\)
30. \(\frac{5a^6}{b^2}\)
   a. \(\frac{10a^{12}}{b^6}\)
   b. \(\frac{25a^{36}}{b^6}\)
   c. \(\frac{25a^{12}}{b^6}\)
   d. \(25a^{12}b^6\)

(5.2) Find the degree of each term.
31. $-5x^3y^3$
32. $10x^3y^2z$
33. $35a^3bc^2$
34. $95xyz$

Find the degree of each polynomial.
35. $y^5 + 7x - 8x^4$
36. $9x^3 + 30y + 25$
37. $-14x^2y - 28x^2y^3 - 42x^2y^2$
38. $6x^3y^2z^2 + 5x^2y^3 - 12xyz$
39. The Glass Bridge Skywalk is suspended 4000 feet over the Colorado River at the very edge of the Grand Canyon. Neglecting air resistance, the height of an object dropped from the Skywalk at time $t$ seconds is given by the polynomial $-16t^2 + 4000$. Find the height of the object at the given times. (See top of next column.)

\[\begin{array}{|c|c|c|c|c|}
\hline
\text{Time (seconds)} & 0 & 1 & 3 & 5 \\
\hline
\text{Height (feet)} & -16t^2 + 4000 & & & \\
\hline
\end{array}\]

\[\Delta 40.\] The surface area of a box with a square base and a height of 5 units is given by the polynomial $2x^2 + 20x$. Fill in the table below by evaluating $2x^2 + 20x$ for the given values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>5.1</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2x^2 + 20x$</td>
<td>40</td>
<td>160</td>
<td>1180</td>
<td>2240</td>
</tr>
</tbody>
</table>

Combine like terms in each expression.
41. $7a^2 - 4a^2 - a^2$
42. $9y + y - 14y$
43. $6a^2 + 4a + 9a^2$
44. $21x^2 + 3x + x^2 + 6$
45. $4a^2b - 3b^2 - 8q^2 - 10a^2b + 7q^2$
46. $2x^{14} + 3x^{13} + 12x^{12} - x^{10}$

Add or subtract as indicated.
47. $(3x^2 + 2x + 6) + (5x^2 + x)$
48. $(2x^2 + 3x^4 + 4x^3 + 5x^2) + (4x^2 + 7x + 6)$
49. $(-5y^2 + 3) - (2y^2 + 4)$
50. $(3x^2 - 7xy + 7y^2) - (4x^2 - xy + 9y^2)$
51. $(-9x^2 + 6x + 2) + (4x^2 - x - 1)$
52. $(8x^4 - 5xy - 10y^2) - (7x^6 - 9xy - 12y^2)$

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TRANSLATING

Perform the indicated operations.
53. Subtract \((3x - y)\) from \((7x - 14y)\).
54. Subtract \((4x^2 + 8x - 7)\) from the sum of \((x^2 + 7x + 9)\) and \((x^2 + 4)\).

(5.3) Multiply each expression.
55. \(4(2a + 7)\)
56. \(9(3a - 3)\)
57. \(-7x(x^2 + 5)\)
58. \(-8y(4y^2 - 6)\)
59. \((3a^3 - 4a + 1)(-2a)\)
60. \((6b^3 - 4b + 2)(7b)\)
61. \((2x + 2)(x - 7)\)
62. \((2x - 5)(3x + 2)\)
63. \((x - 9)^2\)
64. \((x - 12)^2\)
65. \((4a - 1)(a + 7)\)
66. \((6a - 1)(7a + 3)\)
67. \((5x + 2)^2\)
68. \((3x + 5)^2\)
69. \((x + 7)(x^2 + 4x - 5)\)
70. \((x + 2)(x^2 + x + 1)\)
71. \((x^2 + 2x + 4)(x^2 + 2x - 4)\)
72. \((x^3 + 4x + 4)(x^3 + 4x - 4)\)
73. \((x + 7)^3\)
74. \((2x - 5)^3\)

(5.4) Use special products to multiply each of the following.
75. \((x + 7)^2\)
76. \((x - 5)^2\)
77. \((3x - 7)^2\)
78. \((4x + 2)^2\)
79. \((5x - 9)^2\)
80. \((5x + 1)(5x - 1)\)
81. \((7x + 4)(7x - 4)\)
82. \((a + 2b)(a - 2b)\)
83. \((2x - 6)(2x + 6)\)
84. \((4a^2 - 2b)(4a^2 + 2b)\)

Express each as a product of polynomials in \(x\). Then multiply and simplify.

\(\triangle 85\). Find the area of the square if its side is \((3x - 1)\) meters.

\(\triangle 86\). Find the area of the rectangle.

\[(3x - 1)\) meters

\[(5x + 2)\) miles

(5.5) Simplify each expression.
87. \(7^2\)
88. \(-7^2\)
89. \(2x^{-4}\)
90. \((2x)^{-4}\)
91. \(\left(\frac{1}{5}\right)^3\)
92. \(\left(-\frac{2}{3}\right)^2\)
93. \(2^0 + 2^{-4}\)
94. \(6^{-1} - 7^{-1}\)

Simplify each expression. Write each answer using positive exponents only.
95. \(\frac{x^5}{x^3}\)
96. \(\frac{x^4}{x^2}\)
97. \(r^{-3}\)
98. \(y^{-5}\)
99. \(\frac{(bc)^{-2}}{b^c}\)
100. \(\frac{x^{-3}y^{-4}}{x^{-2}y^{-3}}\)
101. \(x^{-4}y^{-6}\)
102. \(\frac{a^2b^{-5}}{a^{-3}b^5}\)
103. \(a^m a^{-5m}\)
104. \(\frac{x^m + h^3}{x^5}\)
105. \((3x^2y^3)^3\)
106. \(a^{m+2} a^{-m+3}\)

Write each number in scientific notation.
107. 0.00027
108. 0.0000084
109. 80,800,000
110. 868,000
111. Google.com is an Internet search engine that handles 2,500,000,000 searches every day. Write 2,500,000,000 in scientific notation. (Source: Google, Inc.)
112. The approximate diameter of the Milky Way galaxy is 150,000 light years. Write this number in scientific notation. (Source: NASA IMAGE/POETRY Education and Public Outreach Program)

Write each number in standard form.
113. \(8.67 \times 10^5\)
114. \(3.86 \times 10^{-3}\)
115. \(8.6 \times 10^{-4}\)
116. \(8.936 \times 10^{-5}\)
117. The volume of the planet Jupiter is 1.43128 \times 10^{15} \text{ cubic kilometers}. Write this number in standard notation. (Source: National Space Science Data Center)
118. An angstrom is a unit of measure, equal to $1 \times 10^{-10}$ meter, used for measuring wavelengths or the diameters of atoms. Write this number in standard notation. (Source: National Institute of Standards and Technology)

Simplify. Express each result in standard form.

119. $(8 \times 10^4)(2 \times 10^{-7})$

120. $\frac{8 \times 10^4}{2 \times 10^{-7}}$

121. $x^2 + 21x + 49$

122. $\frac{5a^3b - 15ab^2 + 20ab}{7a^2}$

123. $(a^2 - a + 4) \div (a - 2)$

124. $(4x^2 + 20x + 7) \div (x + 5)$

125. $\frac{a^3 + a^2 + 2a + 6}{a - 2}$

126. $\frac{9b^3 - 18b^2 + 8b - 1}{3b - 2}$

127. $\frac{4x^3 - 4x^2 + x^2 + 4x - 3}{2x - 1}$

128. $\frac{-10x^2 - x^3 - 21x + 18}{x - 6}$

129. The area of the rectangle below is $(15x^3 - 3x^2 + 60)$ square feet. If its length is $3x^2$ feet, find its width.

Area is $(15x^3 - 3x^2 + 60)$ sq ft

130. The perimeter of the equilateral triangle below is $(21a^3b^5 + 3a - 3)$ units. Find the length of a side.

Perimeter is $(21a^3b^5 + 3a - 3)$ units

MIXED REVIEW

Evaluate.

131. $\left( -\frac{1}{2} \right)^3$

Simplify each expression. Write each answer using positive exponents only.

132. $(4xy^2)(x^3y^3)$

133. $\frac{18x^9}{27x^3}$

134. $\left( \frac{3a^4}{b^2} \right)^3$

135. $(2x^{-4}y^{-3})^{-4}$

136. $a^{-3}b^6$

Perform the indicated operations and simplify.

137. $(6x + 2) + (5x - 7)$

138. $(-y^2 - 4) + (3y^2 - 6)$

139. $(8y^2 - 3y + 1) - (3y^2 + 2)$

140. $(5x^2 + 2x - 6) - (-x - 4)$

141. $4(x(7x^2 + 3))$

142. $(2x + 5)(3x - 2)$

143. $(x - 3)(x^2 + 4x - 6)$

144. $(7x - 2)(4x - 9)$

Use special products to multiply.

145. $(5x + 4)^2$

146. $(6x + 3)(6x - 3)$

Divide.

147. $\frac{8a^4 - 2a^3 + 4a - 5}{2a^2}$

148. $\frac{x^2 + 2x + 10}{x + 5}$

149. $\frac{4x^3 + 8x^2 - 11x + 4}{2x - 3}$
Chapter 5 Test

Evaluate each expression.

1. \(2^5\)  
2. \((-3)^4\)  
3. \(-3^4\)  
4. \(4^{-3}\)

Simplify each exponential expression. Write the result using only positive exponents.

5. \((3^x)^{-5x^y}\)
6. \(\frac{y^2}{y^3}\)
7. \(\frac{5^{-8}}{r^{-3}}\)
8. \(\left(\frac{x^3y^2}{x^3y^4}\right)^2\)
9. \(6x^{-3}y^{-1}\)

Express each number in scientific notation.

10. 563,000  
11. 0.00000863

Write each number in standard form.

12. \(1.5 \times 10^{-3}\)  
13. \(6.23 \times 10^4\)

Simplify. Write the answer in standard form.

14. \((1.2 \times 10^5)(3 \times 10^{-7})\)

15. a. Complete the table for the polynomial \(4xy^2 + 7xyz + x^3y - 2\).

<table>
<thead>
<tr>
<th>Term</th>
<th>Numerical Coefficient</th>
<th>Degree of Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4xy^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7xyz)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(x^3y)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-2)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. What is the degree of the polynomial?


\(5x^2 + 4xy - 7x^2 + 11 + 8xy\)

Perform each indicated operation.

17. \((8x^3 + 7x^2 + 4x - 7) + (8x^3 - 7x - 6)\)
18. \(5x^3 + x^2 + 5x - 2 - (8x^3 - 4x^2 + x - 7)\)
19. Subtract \((4x + 2)\) from the sum of \((8x^2 + 7x + 5)\) and \((x^3 - 8)\).

Multiply.

20. \((3x + 7)(x^2 + 5x + 2)\)
21. \(3x^2(2x^2 - 3x + 7)\)
22. \((x + 7)(3x - 5)\)
23. \(\left(3x - \frac{1}{5}\right)(3x + \frac{1}{5})\)
24. \((4x - 2)^2\)
25. \((8x + 3)^2\)
26. \((x^2 - 9b)(x^2 + 9b)\)

Solve.

27. The height of the Bank of China in Hong Kong is 1001 feet. Neglecting air resistance, the height of an object dropped from this building at time \(t\) seconds is given by
Chapter 5 Cumulative Review

1. Tell whether each statement is true or false.
   a. \(8 \geq 8\)  
   b. \(8 \leq 8\)  
   c. \(23 \leq 0\)  
   d. \(23 \geq 0\)

2. Find the absolute value of each number.
   a. \(|-7.2|\)  
   b. \(|0|\)  
   c. \(|-\frac{1}{2}|\)

3. Divide. Simplify all quotients if possible.
   a. \(\frac{4}{5} \div \frac{5}{16}\)  
   b. \(\frac{7}{10} \div 14\)  
   c. \(\frac{3}{8} \div \frac{3}{10}\)

   a. \(\frac{3}{4} \cdot \frac{7}{21}\)  
   b. \(\frac{1}{2} \cdot \frac{4}{5}\)

5. Evaluate the following.
   a. \(3^2\)  
   b. \(5^3\)  
   c. \(2^4\)
   d. \(7^1\)  
   e. \(\left(\frac{3}{7}\right)^2\)

6. Evaluate \(\frac{2x - 7y}{x^2}\) for \(x = 5\) and \(y = 1\).

7. Add.
   a. \(-3 + (-7)\)  
   b. \(-1 + (-20)\)  
   c. \(-2 + (-10)\)

8. Simplify: \(8 + 3(2 \cdot 6 - 1)\)

9. Subtract 8 from \(-4\).

10. Is \(x = 1\) a solution of \(5x^2 + 2 = x - 8\)?

11. Find the reciprocal of each number.
    a. \(22\)  
    b. \(\frac{3}{16}\)  
    c. \(-10\)  
    d. \(-\frac{9}{13}\)

12. Subtract.
    a. \(-7 - 40\)  
    b. \(-5 - (-10)\)

13. Use an associative property to complete each statement.
    a. \(5 + (4 + 6) = \)  
    b. \((-1 \cdot 2) \cdot 5 = \)  

14. Simplify: \(\frac{4(-3) + (-8)}{5 + (-5)}\)

15. Simplify each expression.
    a. \(10 + (x + 12)\)  
    b. \(-3(x)\)

16. Use the distributive property to write \(-2(x + 3y - z)\) without parentheses.

17. Find each product by using the distributive property to remove parentheses.
    a. \(5(3x + 2)\)  
    b. \(-2(y + 0.3z - 1)\)  
    c. \(-9x + y - 2z + 6)\)

18. Simplify: \(2(6x - 1) - (x - 7)\)

19. Solve \(x - 7 = 10\) for \(x\).

20. Write the phrase as an algebraic expression: double a number, subtracted from the sum of the number and seven

21. Solve: \(\frac{5}{2}x = 15\)

22. Solve: \(2x + \frac{1}{8} = x - \frac{3}{8}\)

23. Solve: \(4(2x - 3) + 7 = 3x + 5\)

24. Solve: \(10 = 5y - 2\)

25. Twice a number, added to seven, is the same as three subtracted from the number. Find the number.

26. Solve: \(\frac{7x + 5}{3} = x + 3\)

27. The length of a rectangular road sign is 2 feet less than three times its width. Find the dimensions if the perimeter is 28 feet.

28. Graph \(x < 5\). Then write the solutions in interval notation.

29. Solve \(F = \frac{9}{5}C + 32\) for \(C\).

30. Find the slope of each line.
    a. \(x = -\)  
    b. \(y = 7\)

31. Graph \(2 < x \leq 4\).
32. Recall that the grade of a road is its slope written as a percent. Find the grade of the road shown.

33. Complete the following ordered pair solutions for the equation $3x + y = 12$.
   a. $(0, )$
   b. $(, 6)$
   c. $(-1, )$

34. Solve the system:
   \[
   \begin{align*}
   3x + 2y &= -8 \\
   2x - 6y &= -9 
   \end{align*}
   \]

35. Graph the linear equation $2x + y = 5$.

36. Solve the system:
   \[
   \begin{align*}
   x &= -3y + 3 \\
   2x + 9y &= 5 
   \end{align*}
   \]

37. Graph the linear equation $x = 2$.

38. Evaluate.
   a. $(−5)^2$
   b. $−5^2$
   c. $2 \cdot 5^2$

39. Find the slope of the line $x = 5$.

40. Simplify: $\frac{(z^2)^3 \cdot z^7}{z^9}$

41. Graph: $x + y < 7$

42. Subtract: $(5y^2 - 6) - (y^2 + 2)$

43. Use the product rule to simplify $(2x^2)(-3x^5)$.

44. Find the value of $−x^2$ when
   a. $x = 2$
   b. $x = -2$

45. Add $(-2x^2 + 5x - 1)$ and $(-2x^2 + x + 3)$.

46. Multiply: $(10x^2 - 3)(10x^2 + 3)$

47. Multiply: $(2x - y)^2$

48. Multiply: $(10x^2 + 3)^2$

49. Divide $6m^2 + 2m$ by $2m$.

50. Evaluate.
   a. $5^{-1}$
   b. $7^{-2}$