

FACTORIZING TRINOMIALS OF THE FORM $ax^2 + bx + c$ BY GROUPING (the $a \cdot c$ Method)

Step 1: Look for a **GCF** and factor it out first.

Step 2: Multiply the coefficient of the leading term **a** by the constant term **c**. List the factors of this product ($a \cdot c$) to find the pair of factors, **f₁** and **f₂**, that sums to **b**, the coefficient of the middle term.

- When **c** is **positive**, factors of $a \cdot c$ have the *same* sign –
 - If the middle term **bx** is *positive*, both factors are *positive*.
 - If the middle term **bx** is *negative*, both factors are *negative*.
 - Find the pair of factors that *adds* to **b**.
- When **c** is **negative**, factors of $a \cdot c$ have *opposite* signs –
 - The *larger* of these factors has the *same* sign as the middle term.
 - Find the pair of factors that *subtracts* to **b**.

Step 3: Rewrite (split) the middle term **bx** using the factors, **f₁** and **f₂**, found in Step 2. The expression now has 4 terms:

$$\begin{aligned} ax^2 + bx + c &= \\ ax^2 + f_1x + f_2x + c & \end{aligned}$$

Step 4: Group the terms of the expression into binomial pairs as shown:

$$(ax^2 + f_1x) + (f_2x + c)$$

Step 5: Factor out a "gcf" from each pair. If the expression can be factored by grouping, the terms will share a common "binomial" factor.

Step 6: Factor out the common binomial factor to write the factorization.

Step 7: Check the result by multiplying.

EXAMPLE: Factor: $8x^3 - 34x^2 + 30x$

Step 1: Factor out the GCF **2x**:

$$8x^3 - 34x^2 + 30x = 2x(4x^2 - 17x + 15)$$

Step 2: Multiply **a • c**. Here, **a = 4**, **c = 15**, and **a • c = 60**. Next, list the factors of **60** to find the pair that *adds* to the middle term **-17x**. Note that the factors of 60 are both *negative*.

<u>Factors of 60</u>	<u>Sum of Factors</u>
-1 • -60	-61
-2 • -30	-32
-3 • -20	-23
-4 • -15	-19
-5 • -12	-17
-6 • -10	-16

Factors of **60** that add to **-17** are **-5** and **-12**.

Step 3: Rewrite the middle term **-17x** using the factors found in Step 2, **-5** and **-12**:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x(4x^2 - 17x + 15) \\ &= 2x(4x^2 - 5x - 12x + 15) \end{aligned}$$

Step 4: Group the terms into binomial pairs as shown:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x(4x^2 - 5x - 12x + 15) \\ &= 2x[(4x^2 - 5x) + (-12x + 15)] \end{aligned}$$

Step 5: Factor out a “gcf” from each group of terms. The gcf of the first group, $(4x^2 - 5x)$, is **x**; the gcf of the second group, $(-12x + 15)$, is **-3**:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x[(4x^2 - 5x) + (-12x + 15)] \\ &= 2x[x(4x - 5) - 3(4x - 5)] \end{aligned}$$

The terms now share a common binomial factor: $(4x - 5)$.

Step 6: Factor out the common binomial factor **(4x - 5)** to write the factorization:

$$\begin{aligned} 8x^3 - 34x^2 + 30x &= 2x[x(4x - 5) - 3(4x - 5)] \\ &= 2x(4x - 5)(x - 3) \end{aligned}$$

Step 7: Check the result by multiplying:

$$\begin{aligned} 2x(4x - 5)(x - 3) &= 2x(4x^2 - 12x - 5x + 15) \\ &= 2x(4x^2 - 17x + 15) \\ &= 8x^3 - 34x^2 + 30x \end{aligned}$$