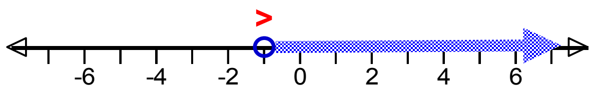
# INEQUALITIES

**Inequalities**, such as $4 < 8$, $x > -1$, and $2x + 10 \ge -12$, are mathematical statements that express the relative order of two expressions. The inequalities $4 < 8$ and $8 > 4$ have the same meaning. To rewrite an inequality, switch the numbers and reverse the direction of the inequality symbol.

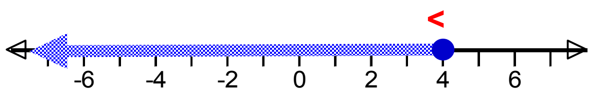
The **solution** of an inequality is the **set** of all numbers that when substituted for the variable makes the inequality a true statement. For example, **2** is a solution of the inequality $\mathbf{x > -1}$ because the statement $\mathbf{2 > -1}$ is true, but $\mathbf{-2}$ is not a solution because the statement $\mathbf{-2 > -1}$ is false.

## GRAPHING INEQUALITIES

One way to describe the solutions of an inequality is to graph the **solution set** on the number line. For example, any number greater than −1 is a solution of the inequality $x > -1$. To graph the solution, we draw an **open circle** " o" at −1 to show that the **endpoint** −1 is not a solution and shade the graph in the direction of the inequality, as shown:




To graph the inequality $y \le 4$, we use a **solid circle** "●" at 4 to show that 4 is a solution:




In general,

1) To graph inequalities with **less than** (<) and **greater than** (>), draw an **open circle "o"** at the endpoint. Shade the graph in the direction of the inequality.

2) To graph the inequalities with **less than or equal to** (≤) or **greater than or equal to** (≥), draw a **solid circle "●"** at the endpoint. Shade the graph in the direction of the inequality.

## SOLVING INEQUALITIES

To solve an inequality, we use the **addition** and **multiplication principles of inequalities** to write the inequality in the form **variable < number** (or >, ≤, ≥). Because an inequality expresses the relative order of two expressions,

1) Multiplying or dividing both sides of an inequality by a **positive number** does not change the solution. For example, when we multiply both sides of the inequality **4 < 8** by **2**, we get a true statement.

$$4 < 8 \qquad \leftarrow \text{true}$$
$$\mathbf{2} \cdot 4 < 8 \cdot \mathbf{2}$$
$$8 < 16 \qquad \leftarrow \text{true}$$

2) Multiplying or dividing both sides of an inequality by a **negative number** creates a false statement. For example, when we multiply both sides of the inequality **4 < 8** by **−2**, we get a false statement. **Reversing the direction of the inequality** makes the statement true.

$$4 < 8 \qquad \leftarrow \text{true}$$
$$\mathbf{-2} \cdot 4 < 8 \cdot \mathbf{-2}$$
$$-8 < -16 \qquad \leftarrow \text{false}$$
$$-8 > -16 \qquad \leftarrow \text{true}$$

**<u>STEPS FOR SOLVING INEQUALITIES</u>**

1) <u>Clear fractions and decimals</u> by multiplying <u>each term</u> of the inequality by the LCD (least common denominator).

2) <u>Remove parentheses</u> by distributing.

3) <u>Combine like terms</u> found on the same side.

4) Use the <u>addition principle</u> to <u>move</u> the variable term to the <u>left</u> side of the inequality so that the inequality reads correctly. Move the number to the <u>right</u> side.

5) Use the <u>multiplication principle</u> to <u>solve</u> for the variable. <u>If the variable term is negative, reverse the direction of the inequality</u>.

6) To <u>check</u> the result, choose any number from the solution set and substitute it for the variable in the original inequality. If a true statement results, you're done!

**<u>EXAMPLE 1</u>:** Solve $2x + 10 \ge -12$

<u>Solution</u>: Use the addition principle to isolate the variable term on the left side of the inequality. Use the multiplication principle to solve for the variable, as shown:

$$2x + 10 \ge -12$$

$$2x + 10 - 10 \ge -12 - 10 \quad \leftarrow \text{isolate the variable term by adding } -10 \text{ to both sides}$$

$$2x \ge -22$$

$$\frac{2x}{2} \ge \frac{-22}{2} \quad \leftarrow \text{divide both sides by 2 to solve for the variable}$$

$$x \ge -11$$

**<u>EXAMPLE 2</u>:** $\frac{2}{3} + \frac{x}{5} < \frac{4}{15}$

<u>Solution</u>: To clear the fractions, multiply each term by the LCD:

$$\frac{2}{3} - \frac{x}{5} < \frac{4}{15}$$

$$^{5}\cancel{15}\left(\frac{2}{\cancel{3}_1}\right) - {}^{3}\cancel{15}\left(\frac{x}{\cancel{5}_1}\right) < {}^{1}\cancel{15}\left(\frac{4}{\cancel{15}_1}\right) \quad \leftarrow \text{multiply each term by the LCD 15 and cancel}$$

$$10 - 3x < 4$$

$$10 - 3x - 10 < 4 - 10 \quad \leftarrow \text{isolate the variable term by adding } -10 \text{ to both sides}$$

$$-3x < -6$$

$$\frac{-3x}{-3} < \frac{-6}{-3} \quad \leftarrow \text{divide both sides by } -3 \text{ then reverse the inequality symbol}$$

$$x > 2$$

Any number greater than 2 is a solution. Using **set builder notation**, the solution set $\{x \mid x > 2\}$ is read "the set of all x such that x is greater than 2."