<u>RULES FOR EXPONENTS</u>

Exponents are used to write repeated multiplication of the same factor. In the exponential expression 2^3, the exponent 3 tells us how many times the base 2 is used as a factor: $2^3 = 2 \cdot 2 \cdot 2$. Similarly, in the expression x^4, the exponent tells us there are four factors of the base x: $x^4 = x \cdot x \cdot x \cdot x$.

When you simplify exponential expressions, it is important to understand the difference between the expressions -3^2 and $(-3)^2$. The *lack* of parentheses in the expression -3^2 means the exponent only raises the base "3" to the 2nd power. So -3^2 evaluates as $-(3 \cdot 3)$. On the other hand, the *use* of parentheses in the expression $(-3)^2$ means the exponent raises the base "−3" to the 2nd power. So $(-3)^2$ evaluates as $(-3) \cdot (-3)$.

1. <u>PRODUCT RULE</u>: $a^m \cdot a^n = a^{m+n}$

To multiply exponential expressions with the same base, <u>keep the base</u>, <u>*add* the exponents</u>.

<u>Examples</u>:

1) $2^3 \cdot 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^7$

We get the same result when we add the powers:

$$2^3 \cdot 2^4 = 2^{3+4} = 2^7$$

2) $p \cdot p^6 \cdot p^8 = p^{1+6+8} = p^{15}$ ← When an exponent is not written, the power is "1".

3) $(-3)^2 \cdot (-3)^6 = (-3)^{2+6} = (-3)^8 = 3^8$ ← A negative number raised to an even power is positive.

4) $-2x^4 (6x)(4x^3) = (-2 \cdot 6 \cdot 4)(x^{4+1+3}) = -48x^8$ ← Use the associative property to multiply the coefficients and add the variable powers.

2. <u>QUOTIENT RULE</u>: $\frac{a^m}{a^n} = a^{m-n}$

To divide exponential expressions with the same base, <u>keep the base</u>, <u>subtract the exponents</u>.

a) When the <u>power in the numerator is greater</u> than the power in the denominator (*top heavy*), the <u>remainder goes in the numerator</u>.

<u>Examples</u>:

1) $\frac{2^5}{2^2} = \frac{\not{2} \cdot \not{2} \cdot 2 \cdot 2 \cdot 2}{\not{2} \cdot \not{2}} = \frac{2^3}{1} = 2^3 = 8$

Notice, we get the same result when we subtract the power in the denominator from the power in the numerator:

$$\frac{2^5}{2^2} = 2^{5-2} = 2^3 = 8$$

2) $\frac{(-5)^7}{(-5)^4} = (-5)^{7-4} = (-5)^3 = -125$ ← A negative number raised to an odd power is negative.

3) $\frac{x^{12}}{x^7} = x^{12-7} = x^5$

4) $\frac{9x^4y^5}{3x^2y} = \frac{\cancel{9}x^{4-2}y^{5-1}}{\cancel{3}} = \frac{3x^2y^4}{1} = 3x^2y^4$

b) When the <u>power in the denominator is greater</u> than the power in the numerator (*bottom heavy*), the <u>remainder goes in the denominator</u> and <u>a "1" goes in the numerator when necessary</u>.

<u>Examples</u>:

1) $\frac{2^4}{2^6} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2} = \frac{1}{2^2} = \frac{1}{4}$

We get the same result when we subtract the power in the numerator from the power in the denominator:

$\frac{2^4}{2^6} = \frac{1}{2^{6-4}} = \frac{1}{2^2} = \frac{1}{4}$

2) $\frac{(-4)^3}{(-4)^8} = \frac{1}{(-4)^{8-3}} = \frac{1}{(-4)^5}$

3) $\frac{k^7}{k^{11}} = \frac{1}{k^{11-7}} = \frac{1}{k^4}$

4) $-\frac{7m^2n^5}{28m^8n^{10}} = -\frac{\cancel{7}}{\cancel{28}m^{8-2}n^{10-5}} = -\frac{1}{4m^6n^5}$

5) $\frac{36x^2y^3}{12x^4y} = \frac{\cancel{36}y^{3-1}}{\cancel{12}x^{4-2}} = \frac{3y^2}{1 \cdot x^2} = \frac{3y^2}{x^2}$ ← The x's are *bottom heavy*; the y's are *top heavy*.

3. **<u>POWER RULE:</u> $(a^m)^n = a^{m \cdot n}$**

To raise an exponential expression to a power, keep the base, multiply the exponents.

Examples:

1) $\left(2^4\right)^3 = 2^4 \cdot 2^4 \cdot 2^4 = 2^{4+4+4} = 2^{12}$

We get the same result when we multiply the powers:

$\left(2^4\right)^3 = 2^{4 \cdot 3} = 2^{12}$

2) $\left(-7^3\right)^5 = (-7)^{3 \cdot 5} = (-7)^{15} = -7^{15}$

3) $\left(x^2\right)^4 = x^{2 \cdot 4} = x^8$

4. **POWER RULES FOR PRODUCTS AND QUOTIENTS**

a) **Power Rule for Products: $(ab)^n = a^n b^n$**

To raise a product to a power, distribute the power to each base and multiply the exponents.

Examples:

1) $\left(x^4 y\right)^3 = x^{4 \cdot 3} y^{1 \cdot 3} = x^{12} y^3$

2) $\left(-2p^4 q r^2\right)^4 = (-2)^{1 \cdot 4} p^{4 \cdot 4} q^{1 \cdot 4} r^{2 \cdot 4} = (-2)^4 p^{16} q^4 r^8 = 16p^{16} q^4 r^8$

Think: “a negative raised to an even power is always positive.”

b) **Power Rule for Quotients: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$**

To raise a quotient to a power, distribute the power to the numerator and the denominator and multiply the powers.

Examples:

1) $\left(\frac{5}{x^2}\right)^4 = \frac{5^4}{x^8} = \frac{625}{x^8}$

2) $\left(\frac{-2x^2 y}{z^3}\right)^2 = \frac{2^2 x^4 y^2}{z^6} = \frac{4x^4 y^2}{z^6}$

5. EXPONENTS OF "1" AND "0"

a) POWER OF "1:" $a^1 = a$

Any number or variable raised to the 1st power is the number or variable.

Examples:

1) $3^1 = 3$ 2) $x^1 = x$ 3) $(ab)^1 = a^1 b^1 = ab$

b) POWER OF "0:" $a^0 = 1$

Any number or variable raised to the zero power always equals 1.

When a number or variable is divided by itself, the result is 1. For example,

$\frac{4}{4} = 1;$ $\frac{-3}{-3} = 1;$ $\frac{y}{y} = 1;$ $\frac{-2t}{-2t} = 1;$

We get the same result when we apply the quotient rule:

$$\frac{2^5}{2^5} = 2^{5-5} = 2^0 = 1$$

Examples:

1) $5^0 = 1$ 2) $x^0 = 1$

3) $-2y^0 = -2(1) = -2$ 4) $-4^0 - 5^0 = -1 - 1 = -2$

5) $j^4 \cdot j^0 \cdot j^5 = j^{4+0+5} = j^9$

6. RULE FOR NEGATIVE EXPONENTS; $a^{-n} = \frac{1}{a^n}$

A negative exponent means "take the reciprocal of." To simplify negative exponents, take the reciprocal of the base (flip) and make the power positive.

For example, 2^{-3} is read "take the reciprocal of 2^3." Since $2^3 = 8$, its reciprocal is $\frac{1}{2^3}$, or $\frac{1}{8}$:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

a) If the *power in the numerator is negative*, move it to the denominator, and make the power positive.

Examples:

1) $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

2) $y^{-5} = \frac{1}{y^5}$

3) $\frac{x^{-2}}{y} = \frac{1}{x^2 y}$

4) $2a^3b^{-4} = \frac{2a^3b^{-4}}{1} = \frac{2a^3}{b^4}$

b) If the *power in the denominator is negative*, move it to the numerator, and make the power positive.

Examples:

1) $\frac{1}{5^{-2}} = 5^2 = 25$

2) $\frac{1}{p^{-7}} = p^7$

3) $\frac{x^5}{y^{-3}} = \frac{x^5y^3}{1} = x^5y^3$

4) $\frac{5}{m^{-3}n^{-2}} = \frac{5m^3n^2}{1} = 5m^3n^2$

c) If the *numerator and the denominator have negative factors*, simplify the negative powers first. Move negative powers in the numerator to the denominator; move negative powers in the denominator to the numerator.

Examples:

1) $\frac{2^{-4}}{3^{-2}} = \frac{3^2}{2^4} = \frac{9}{16}$

2) $\frac{x^{-7}}{y^{-8}} = \frac{y^8}{x^7}$

3) $\frac{3^{-2}xy^{-3}}{x^{-4}y^3} = \frac{x \bullet x^4}{3^2y^3 \bullet y^3} = \frac{x^5}{9y^6}$

Simplify negative exponents first, and then apply the product and quotient rules as needed.

4) $\left(\frac{5p^2}{q^3}\right)^{-2} = \left(\frac{q^3}{5p^2}\right)^2 = \frac{q^6}{5^2p^4} = \frac{q^6}{25p^4}$

Taking the reciprocal of the inside expression ("flipping it") makes the outside power positive.

5) $\frac{-2p^{-4}}{(p^3)^{-2}} = \frac{-2p^{-4}}{p^{-6}} = \frac{-2p^6}{p^4} = -2p^2$ ← Because −2 is not raised to a power, the result is negative.