STRATEGIES FOR FINDING THE LEAST COMMON MULTIPLE (LCM)/LEAST COMMON DENOMINATOR (LCD)

The **least common multiple (LCM)** of a given set of numbers is the *smallest* positive number divisible by the numbers in the set. For example, if we list the multiples of 4 and 6, we can see these numbers share common multiples of 12, 24, 36, and 48 to name a few.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, …

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, …

Even though 24, 36 and 48 are multiples of 4 and 6, the **LCM** is 12 because 12 is the *smallest* number divisible by 4 and 6.

When we need to find a common denominator for a given set of fractions, the **LCM** is called the **least common denominator (LCD)**. To find the **LCD** of a given set of fractions, check the denominators of the fractions:

STRATEGIES

1) *Do the smaller denominators divide the larger?* If they do, the larger denominator is the LCD.

EXAMPLE 1: Find the LCD of $\frac{3}{4}, \frac{1}{2}, \text{and } \frac{5}{8}$

Because 8 is divisible by 4 and 2, the LCD = 8.

2) *Are the denominators prime or relatively prime numbers?* (Prime numbers are numbers divisible only by themselves and 1; relatively prime numbers share no common factor.) When the denominators are prime or relatively prime, multiply the denominators to find the LCD.

EXAMPLE 2: Find the LCD of $\frac{2}{3}, \frac{4}{5}, \text{and } \frac{1}{2}$

The denominators of the fractions are *prime* numbers. To find the LCD, multiply the denominators:

$$\text{LCD} = 2 \cdot 3 \cdot 5 = 30.$$

EXAMPLE 3: Find the LCD of $\frac{3}{4} \text{ and } \frac{5}{7}$

The denominators of the fractions are *relatively prime* numbers because they share no common factors: $4 = 2 \cdot 2$ and $7 = 1 \cdot 7$. To find the LCD, multiply the denominators:

$$\text{LCD} = 4 \cdot 7 = 28.$$

EXAMPLE 4: Find the LCD of $\frac{2}{x}$ and $\frac{4}{9}$

Because the value of "x" is unknown, the only factors of x are "1" and "x." This means that 9 and "x" share no common factors, so the LCD = 9 • x.

3) If the largest denominator is *not divisible* by the smaller denominators, list the multiples of the *largest* to find the LCD.

EXAMPLE 5: Find the LCD of $\frac{5}{4}, \frac{1}{6}, \frac{7}{10}$, and $\frac{8}{15}$

The smaller denominators do not divide the larger. As shown below, we find the LCD sooner when we list the multiples of the largest denominator.

Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, **60**, …

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 64, **60**, …

Multiples of 10: 10, 20, 30, 40, 50, **60**, …

Multiples of 15: 15, 30, 45, **60**, …

LCD = 60

4) If the LCD is not among the first 5 or 6 multiples you list, try prime factorization and a factor box.

EXAMPLE 6: Find the LCD of $\frac{5}{12}, \frac{2}{15}$, and $\frac{7}{18}$

Step 1: Write the prime factorization of each denominator and list the factors in a table of primes, as shown:

	2	**3**	**5**
12 = 2 • 2 • 3 = 2^2 • 3 →	2^2	3	
15 = 3 • 5 →		3	5
18 = 2 • 3 • 3 = 2 • 3^2 →	2	3^2	

Step 2: Take the *highest power* of any factor the numbers share in common and any factor the numbers do not share in common. The LCD is the product of these factors:

LCD = $2^2 \cdot 3^2 \cdot 5$ = 180