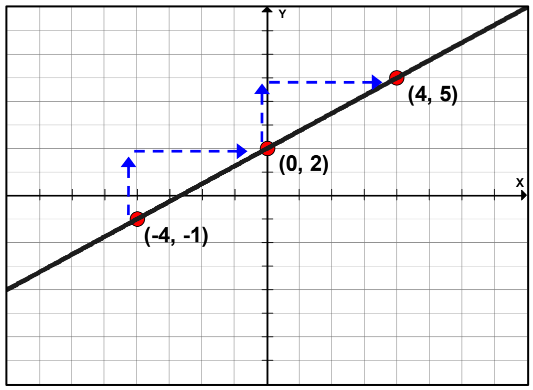
THE SLOPE OF A LINE

Consider the line containing the points (−4, −1), (0, 2) and (4, 5). When we move from the point (−4, −1) to the point (0, 2) the y-coordinate increases by 3 units; the x-coordinate increases by 4 units.



This pattern repeats when we move from the point (0, 2) to the point (4, 5). The vertical change or **rise** between any two points, such as (0, 2) and (4, 5), is the difference of the y-coordinates: 5 – 2 = 3. The horizontal change or **run** is the difference of the x-coordinates: 4 – 0 = 4. The ratio of the change in the y to the change in x is called the **slope** of the line:

$$\text{slope} = \frac{\text{change in y}}{\text{change in x}} = \frac{\text{rise}}{\text{run}} = \frac{5-2}{4-0} = \frac{3}{4}$$

Finding Slope When Two Ordered Pairs are Given

Given any two points on a line, we'll call them (x_1, y_1) and (x_2, y_2), we can find the line's slope using the following formula:

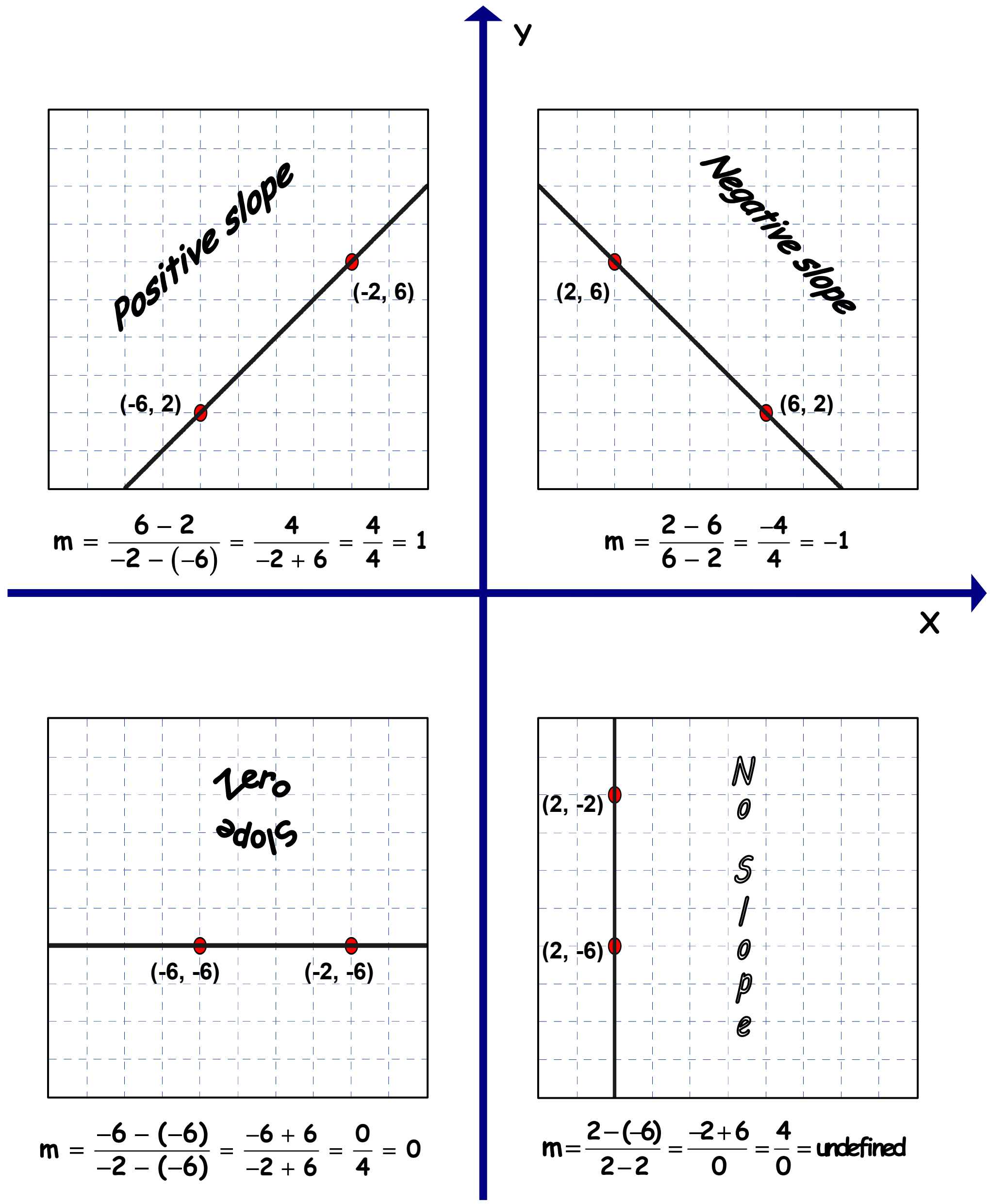
$$\text{slope} = m = \frac{\text{change in y}}{\text{change in x}} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

<u>Example 1</u>: Find the slope of the line passing through the points (−6, −2) and (−2, 4).

To find the slope, let $(x_1, y_1) = (-6, -2)$ and $(x_2, y_2) = (-2, 4)$. Then substitute the values for x_1, y_1 and x_2, y_2 in the formula and simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{-2-(-6)} = \frac{4+2}{-2+6} = \frac{6}{4} = \frac{3}{2}$$

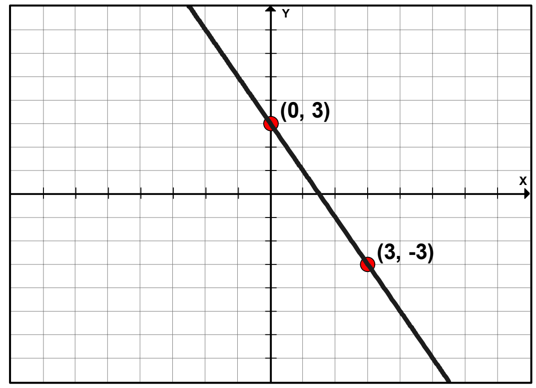
The slope of a line determines its slant. When the slope is positive, the line slants upward; when the slope is negative the line slants downward.



Finding Slope When an Equation is Given

The equation **y = mx + b** is called the slope-intercept form of a linear equation because the **slope** is **m**, the coefficient of the x term, and the y-intercept is the point **(0, b)**. To see this, consider the graph of the equation y = −2x + 3. The line contains the points (0, 3) and (3, −3), as the table shows.

X	y = −2x + 3	Y
0	y = −2(0) + 3 = 3	3
1	y = −2(1) + 3 = 3	1
2	y = −2(2) + 3 = −1	−1
3	y = −2(3) + 3 = −3	−3



If we let $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (3, -3)$ and substitute these values into the formula, we get:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-3}{3-0} = \frac{-6}{3} = -2$$

The slope m = −2 and the y-intercept = (0, 3)

If the given equation is written in the general form ax + by = c, solve the equation for y to write it in the form y = mx + b.

<u>Example 2</u>: Find the slope and y-intercept of the equation 4x – 3y = 12.

To find the slope and y-intercept, solve the equation for y:

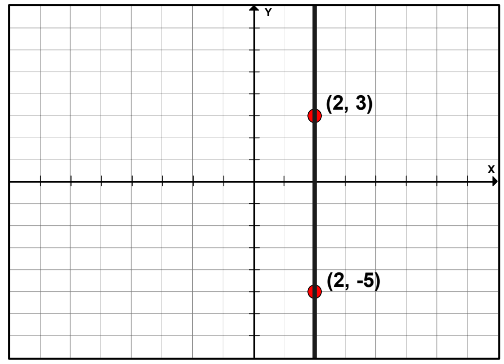
$$4x - 3y = 12$$
$$4x - 3y - 4x = -4x + 12$$
$$-3y = -4x + 12$$
$$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{12}{-3}$$
$$y = \frac{4}{3}x - 4$$

The slope **m** = $\frac{4}{3}$, the coefficient of the x term and the **y-intercept** = (0, −4).

Vertical Lines: x = a

The graph of the equation x = 2 contains the points (2, 3) and (2, −5), as the table shows.

X	Y
2	−5
2	0
2	3



If we let (x_1, y_1) = (2, 3) and (x_2, y_2) = (2, −5) and substitute these values into the formula we get:

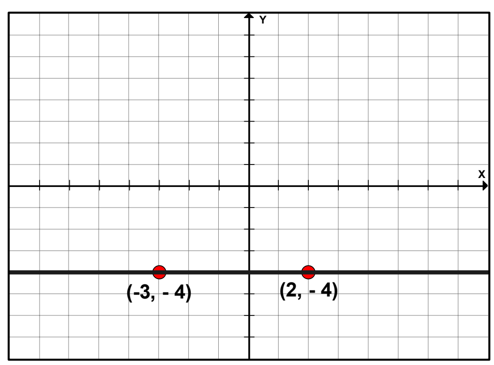
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-3}{2-2} = \frac{-8}{0} = \text{undefined}$$

In general, because the x-coordinates are the same, the slope of a vertical line **x = a** is always **undefined**.

Horizontal Lines: y = b

The graph of the equation y = −4 contains the points (−3, −4) and (2, −4), as the table shows:

X	Y
−3	−4
0	−4
2	−4



If we let (x_1, y_1) = (−3, −4) and (x_2, y_2) = (2, −4) and substitute these values into the formula we get:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-(-4)}{2-(-3)} = \frac{-4+4}{2+3} = \frac{0}{5} = 0$$

In general, because the y-coordinates are the same, the slope of any horizontal line **y = b** is always **0**.