

## ALGEBRAIC EXPRESSIONS

An **algebraic expression** is a collection of numbers and variables separated by operation signs and grouping symbols. Examples of algebraic expressions include:

$$-2, \quad -x, \quad 3y + 5, \quad x - y + z, \quad \text{and} \quad 2x - y^2$$

In the expression " $3y + 5$ ,"  $3y$  and  $5$  are called the **terms** of the expression;  $3y$  is called the **variable term**;  $5$  is called the **constant term**. Variable terms have two parts – a numerical part (the number), called the **coefficient**, and a literal part (the letter or **variable**). The term  $3y$  is read "3 times  $y$ ." Similarly, the expression " $-x$ " is read " $-1 \cdot$  times  $x$ ."

**To evaluate an algebraic expression,**

**Step 1:** Substitute the given values for the variables into the expression. Use parentheses or brackets to substitute negative numbers! Be careful to keep the operations that are in the original problem.

**Step 2:** Simplify the expression by following the **Order of Operations**:

- **Parentheses** – perform all operations inside parentheses, brackets and absolute value symbols.
- **Exponents** – evaluate any expressions with exponents.
- **Multiply/Divide** – do all multiplications and divisions as they appear, in order from left to right.
- **Add/Subtract** – do all additions and subtractions as they appear, in order from left to right.

**EXAMPLE 1:** Evaluate  $2x^2 - 5xy - y^3$  for  $x = -3$  and  $y = -2$

To evaluate the expression, substitute the values for  $x$  and  $y$ , and then simplify using the order of operations.

$$\begin{aligned} 2x^2 - 5xy - y^3 &= 2(-3)^2 - 5(-3)(-2) - (-2)^3 && \leftarrow \text{simplify powers} \\ &= 2(9) - 5(-3)(-2) - (-8) && \leftarrow \text{multiply left to right; change the double} \\ &= 18 + 15(-2) + 8 && \text{negative sign to a positive sign} \\ &= 18 - 30 + 8 = -12 + 8 = -4 && \leftarrow \text{add and subtract left to right} \end{aligned}$$

**EXAMPLE 2:** Evaluate  $3|x + y|^2$  for  $x = 5$  and  $y = -10$

In the order of operations, absolute value has the same priority as parentheses.

$$\begin{aligned} 3|x + y|^2 &= 3|5 + (-10)|^2 && \leftarrow \text{simplify inside the absolute value} \\ &= 3|-5|^2 && \leftarrow \text{take the absolute value} \\ &= 3(5)^2 && \leftarrow \text{evaluate the power} \\ &= 3(25) = 75 && \leftarrow \text{multiply} \end{aligned}$$

**EXAMPLE 3:** Evaluate  $\frac{10x-5y}{x+y}$  for  $x = -4$  and  $b = -2$

Substitute the values for  $x$  and  $y$ . Then simplify the numerators and denominators using the order of operations.

$$\begin{aligned} \frac{10x-5y}{x+y} &= \frac{10(-4)-5(-2)}{(-4)+(-2)} && \leftarrow \text{multiply left to right} \\ &= \frac{-40+10}{-6} && \leftarrow \text{combine } -4 \text{ and } -2 \\ &= \frac{-30}{-6} = 5 && \leftarrow \text{combine } -40 \text{ and } 10 \\ & && \leftarrow \text{divide} \end{aligned}$$

**To simplify an algebraic expression,**

**Step 1:** Use the distributive law to remove parentheses and grouping symbols.

**Step 2:** Combine like terms. Like terms have the same variables raised to the same powers.

**EXAMPLE 1:** Simplify  $6x - 7 - 4(2x - 5)$

Solution: Begin by removing the parentheses. Then combine like terms.

$$\begin{aligned} 6x - 7 - 4(2x - 5) &= && \leftarrow \text{distribute } -4 \\ 6x - 7 - 8x + 20 &= && \leftarrow \text{combine like terms} \\ -2x + 13 & && \leftarrow \text{done!} \end{aligned}$$

**EXAMPLE 2:** Simplify  $-5(10r + 4) + 8(2r - 3)$

Solution: Remove each set of parentheses by distributing left to right. Then combine.

$$\begin{aligned} -5(10r + 4) + 8(2r - 3) &= && \leftarrow \text{distribute } -5 \text{ and } 8 \\ -50r - 25 + 16r - 24 &= && \leftarrow \text{combine like terms} \\ -34r - 49 & && \leftarrow \text{done} \end{aligned}$$

**EXAMPLE 3:** Simplify  $2x - 3[5x - 2(6 - x)]$

Solution: To simplify expressions with nested parentheses, begin by simplifying the innermost set.

$$\begin{aligned} 2x - 3[5x - 2(6 - x)] &= && \leftarrow \text{distribute } -2 \\ 2x - 3[5x - 12 + 2x] &= && \leftarrow \text{combine } 5x \text{ and } 2x \\ 2x - 3[7x - 12] &= && \leftarrow \text{distribute } -3 \\ 2x - 21x + 36 &= && \leftarrow \text{combine } 2x \text{ and } -21x \\ -19x + 36 & && \leftarrow \text{done!} \end{aligned}$$