ALGEBRAIC EXPRESSIONS

An algebraic expression is a collection of numbers and variables separated by operation signs and grouping symbols. Examples of algebraic expressions include:

\[-2, \quad -x, \quad 3y + 5, \quad x - y + z, \quad \text{and} \quad 2x - y^2\]

In the expression "3y + 5," 3y and 5 are called the terms of the expression; 3y is called the variable term; 5 is called the constant term. Variable terms have two parts – a numerical part (the number), called the coefficient, and a literal part (the letter or variable). The term 3y is read "3 times y." Similarly, the expression "−x" is read "−1 • times x."

To evaluate an algebraic expression,

**Step 1:** Substitute the given values for the variables into the expression. Use parentheses or brackets to substitute negative numbers! Be careful to keep the operations that are in the original problem.

**Step 2:** Simplify the expression by following the Order of Operations:

- Parentheses – perform all operations inside parentheses, brackets and absolute value symbols.
- Exponents – evaluate any expressions with exponents.
- Multiply/Divide – do all multiplications and divisions as they appear, in order from left to right.
- Add/Subtract – do all additions and subtractions as they appear, in order from left to right.

**EXAMPLE 1:** Evaluate $2x^2 - 5xy - y^3$ for $x = -3$ and $y = -2$

To evaluate the expression, substitute the values for $x$ and $y$, and then simplify using the order of operations.

\[
2(-3)^2 - 5(-3)(-2) - (-2)^3 = 2(9) - 5(-3)(-2) - (-8) = 18 + 15(2) + 8 = 18 - 30 + 8 = -12 + 8 = -4
\]

**EXAMPLE 2:** Evaluate $3| x + y |^2$ for $x = 5$ and $y = -10$

In the order of operations, absolute value has the same priority as parentheses.

\[
3| x + y |^2 = 3| 5 + (-10) |^2 = 3| -5 |^2 = 3(5)^2 = 3(25) = 75
\]
EXAMPLE 3: Evaluate \( \frac{10x - 5y}{x + y} \) for \( x = -4 \) and \( b = -2 \)

Substitute the values for \( x \) and \( y \). Then simplify the numerators and denominators using the order of operations.

\[
\frac{10x - 5y}{x + y} = \frac{10(-4) - 5(-2)}{(-4) + (-2)} \quad \text{← multiply left to right}
\]

\[
= \frac{-40 + 10}{-6} \quad \text{← combine -4 and -2}
\]

\[
= \frac{-30}{-6} = 5 \quad \text{← divide}
\]

To simplify an algebraic expression,

**Step 1:** Use the distributive law to remove parentheses and grouping symbols.

**Step 2:** Combine like terms. Like terms have the same variables raised to the same powers.

**EXAMPLE 1:** Simplify \( 6x - 7 - 4(2x - 5) \)

Solution: Begin by removing the parentheses. Then combine like terms.

\[
6x - 7 - 4(2x - 5) = \quad \text{← distribute -4}
\]

\[
= 6x - 7 - 8x + 20 \quad \text{← combine like terms}
\]

\[
= -2x + 13 \quad \text{← done!}
\]

**EXAMPLE 2:** Simplify \( -5(10r + 4) + 8(2r - 3) \)

Solution: Remove each set of parentheses by distributing left to right. Then combine.

\[
-5(10r + 5) + 8(2r - 3) = \quad \text{← distribute -5 and 8}
\]

\[
= -50r - 25 + 16r - 24 \quad \text{← combine like terms}
\]

\[
= -34r - 49 \quad \text{← done}
\]

**EXAMPLE 3:** Simplify \( 2x - 3[5x - 2(6 - x)] \)

Solution: To simplify expressions with nested parentheses, begin by simplifying the innermost set.

\[
2x - 3\left[5x - 2(6 - x)\right] = \quad \text{← distribute -2}
\]

\[
= 2x - 3[5x - 12 + 2x] \quad \text{← combine 5x and 2x}
\]

\[
= 2x - 3[7x - 12] \quad \text{← distribute -3}
\]

\[
= 2x - 21x + 36 \quad \text{← combine 2x and -21x}
\]

\[
= -19x + 36 \quad \text{← done!}
\]