

Divisibility Rules

Which of the following fractions can be written in lowest terms?

i) $\frac{117}{225}$ ii) $\frac{27}{42}$ iii) $\frac{51}{85}$ iv) $\frac{128}{131}$

Divisibility rules can be used to determine whether or not a fraction needs to be reduced. The rules are based on the patterns that occur when we list the multiples of any number. For example, when we list the multiples of 2, we get even numbers – numbers that end in 0, 2, 4, 6, or 8.

Multiples of 2: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

This means that all even numbers share a common factor of 2. So if the numerator and the denominator of a fraction are both even numbers, such as $\frac{4}{6}$, we can reduce the fraction by canceling the common factor of 2:

$$\frac{4}{6} = \frac{\cancel{2} * 2}{\cancel{2} * 3} = \frac{2}{3}$$

Other divisibility rules are listed below. After you have read the rules, use them to determine which of the fractions above is written in lowest terms and which can be simplified. Simplify any fractions that are not written in lowest terms.

<u>Number</u>	<u>Rule</u>
0	Division by 0 is undefined – for example, $\frac{5}{0}$ = undefined .
1	Every number is divisible by 1
2	If a number ends in 0, 2, 4, 6, or 8 (even), the number is divisible by 2.
3	If the sum of a number's digits is a multiple of 3, the number is divisible by 3. For example, 3 divides 18. When we add the digits of 18, we get a multiple of 3. This tell us we can divide the number by 3. $18 \rightarrow 1 + 8 = 9.$
4	If the last two digits of a number are divisible by 4, the number is divisible by 4. If the last two digits of a number are 0's, the number is divisible by 4 because 4 divides 100. For example, 324 is divisible by 4 because 4 divides 24, and 1500 is divisible by 4 because the last two digits are 0's.
5	If a number ends in 0 or 5, the number is divisible by 5.

Number**Rule**

7

When we list the multiples of 7, we get

Multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, ...

As you can see, we need to list 10 multiples of 7 before the one's digit repeats. This makes the rule for 7 somewhat complicated. Rather than use the rule, divide the number by 7.

8

If the last three digits of a number are divisible by 8, the number is divisible by 8. If the last three digits are 0's the number is divisible by 8 because 8 divides 1000.

9

If the sum of a number's digits is a multiple of 9, the number is divisible by 9.

10

Any number ending in 0 is divisible by 10.

11

If all of the digits of a number are the same and the number of digits is even, the number is divisible by 11. For example, 11 divides 66 and 2222. But, 11 does not divide 777 because the number of digits is odd.

Divisibility rules can be combined to form divisibility rules for larger numbers. For example, the rules for 2 and 3 can be combined to form a rule for 6 because $6 = 2 * 3$.

Rule for 6: If a number is divisible by 2 and 3 the number is divisible by 6. This means 6 will divide any *even* number whose digits sum to a multiple of 3.

Similarly, the rules for 3 and 4 can be combined to form a rule for 12 because $12 = 3 * 4$.

Rule for 12: If a number is divisible by 3 and 4, the number is divisible by 12.

How many rules can you form?

Answers:

i) $\frac{117}{225} = \frac{\cancel{9} * 13}{\cancel{9} * 25} = \frac{13}{25}$; ii) $\frac{27}{42} = \frac{3 * 9}{3 * 14} = \frac{9}{14}$; iii) $\frac{51}{85} = \frac{3 * 17}{5 * 17} = \frac{3}{5}$; iv) lowest terms