RATIO AND PROPORTION

RATIOS AND RATES

Quantities such as 8 feet, 16 cents or 10 hours are numerical quantities written with units. A **ratio** is a comparison of two quantities with the **same** units. For example, if we want to compare the heights of two trees, one 6 feet tall and the other 8 feet tall, we can write this ratio three ways:

1) As a **fraction**: $\frac{6\ \text{ft}}{8\ \text{ft}} = \frac{6}{8} = \frac{2 \cdot 3}{2 \cdot 4} = \frac{3}{4}$

2) With a **colon**: 6 ft **:** 8 ft = 6 **:** 8 = 3 **:** 4

3) With the word **to**: 6 ft to 8 ft. = 6 to 8 = 3 to 4.

Notice that in each case,

- The **order** of the numbers in a ratio is important! To write a ratio as a fraction, place the **first** number of the ratio in the **numerator**; place the **second** number in the **denominator**.
- The ratio is written in lowest terms.
- The units are not written as part of the ratio. Because a ratio compares two quantities with the **same** units – convert the units if they are different.

Example 1: Write each ratio as a fraction in lowest terms.

A) 15 pounds to 24 pounds

The units are the same, so we can write this ratio as

15 pounds to 24 pounds = $\frac{15}{24} = \frac{3 \cdot 5}{3 \cdot 8} = \frac{5}{8}$

B) 75 cents to $1.25

Here, the units are not the same. Since $1 = 100 cents, to convert $1.25 to cents, drop the dollar sign and move the decimal point two places to the right.

$1.25 = 1.25 = 125 cents

75 cents to $1.25 = 75 cents to 125 cents = $\frac{75}{125} = \frac{3 \cdot 25}{5 \cdot 25} = \frac{3}{5}$

A **rate** is a comparison of two quantities with **different** units, such as 10 g per 180 mL. Like a ratio, a rate can be written as a fraction, with a colon, or with the word to. A rate is also expressed in lowest terms. Unlike a ratio, the units are written as part of the rate. For example, to write the rate "10 g per 180 mL" as a fraction in lowest terms, cancel the corresponding 0's and keep the units:

10 g per 180 mL = $\frac{10\ \text{g}}{180\ \text{mL}} = \frac{1\ \text{g}}{18\ \text{mL}}$

PROPORTIONS

A **proportion** is a mathematical statement that two ratios or rates are equal. For example, whenever we write equivalent fractions, we create a proportion, such as the one shown below:

$$\frac{3}{4} = \frac{6}{8}$$

In a true proportion, the **cross products** are **equal**:

$$\frac{3}{4} = \frac{6}{8} \qquad \begin{aligned} 3(8) &= 24 \\ 4(6) &= 24 \end{aligned}$$

Because the cross products are equal, we can solve a proportion when one of the numbers is unknown.

Example 2: Solve $\frac{4}{9} = \frac{12}{x}$

To solve the proportion 1) cross multiply the ratios, 2) write an equation; and 3) solve for the variable.

$$\frac{4}{9} = \frac{12}{x}$$ ← Cross multiply the ratios

$$4 \bullet x = 9 \bullet 12$$ ← Write an equation

$$4x = 108$$

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{108}{4}$$ ← Solve the equation for x

$$x = 27$$

Example 3: Solve $\frac{\frac{4}{5}}{\frac{2}{7}} = \frac{x}{\frac{3}{4}}$

To solve the proportion, begin by finding the cross products:

$$\frac{\frac{4}{5}}{\frac{2}{7}} = \frac{x}{\frac{3}{4}}$$

$$\frac{2}{7} \bullet x = \frac{\cancel{4}}{5} \bullet \frac{3}{\cancel{4}}$$ ← Cancel common factors

$$\frac{2}{7}x = \frac{3}{5}$$

$$\frac{7}{2} \bullet \frac{2}{7}x = \frac{3}{5} \bullet \frac{7}{2}$$ ← Multiply both sides by the reciprocal of 2/7

$$x = \frac{21}{10}$$

Solving Problems with Proportions

Proportions are often used to solve a variety of problems, such as estimating wildlife populations, scaling distances on a map, or calculating mixtures and dosages.

Example 5: Solve the problem by writing a proportion:

"The brewing directions on a bag of ground coffee recommend 2 tablespoons (tbs) of coffee for every 6 ounces (oz) of water. If the average-size coffee cup holds 6 ounces of coffee, how many tablespoons of coffee are needed to make eight 6-oz cups?"

To write the proportion,

1) **Identify the given ratio or rate and write it as a fraction**. In the problem above, we are given the rate "2 tablespoons of coffee for every 6 ounces of water." We can write this rate as

$$\text{2 tbs to 6 oz} = \frac{2 \text{ tbs}}{6 \text{ oz}} = \frac{1 \text{ tbs}}{3 \text{ oz}}$$

2) **Assign a variable to represent the unknown quantity then write the second ratio or rate as a fraction**. Let x = the number of tablespoons needed to make eight 6-oz cups. Given that each coffee cup holds 6 ounces, we need 48 ounces of water to make eight 6-oz cups of coffee.. This gives us the rate:

$$\text{x tbs to 48 oz} = \frac{x \text{ tbs}}{48 \text{ oz}}$$

3) **Write the proportion**. Set the first ratio equal to the second. Note that, when we write a proportion, we are making a statement that the first ratio has the same value as the second. For this reason, the order in which we write a proportion is important! **Always place like units across from each other**, as shown:

$$\frac{1 \text{ tbs}}{3 \text{ oz}} = \frac{x \text{ tbs}}{48 \text{ oz}} \quad \begin{matrix} \leftarrow \text{tbs across from tbs} \\ \leftarrow \text{oz across from oz} \end{matrix} \quad \text{or} \quad \frac{3 \text{ oz}}{1 \text{ tbs}} = \frac{48 \text{ oz}}{x \text{ tbs}} \quad \begin{matrix} \leftarrow \text{oz across from oz} \\ \leftarrow \text{tbs across from tbs} \end{matrix}$$

Because the cross products are equal, inverting the ratios gives us an equivalent proportion.

4) **Solve the proportion**. To find the number of tablespoons, drop the units, cross multiply, and solve the resulting equation:

$$\frac{1}{3} = \frac{x}{48}$$

$$3x = 48$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{48}{3}$$

$$x = 16 \text{ tbs}$$

Hint: When you solve a problem with a proportion, you may find it helpful to state the problem in the following way: 2 tbs is to 6 oz **as** x tbs is to 48 oz.