

## RATIO AND PROPORTION

### RATIOS AND RATES

Quantities such as 8 feet, 16 cents or 10 hours are numerical quantities written with units. A **ratio** is a comparison of two quantities with the **same** units. For example, if we want to compare the heights of two trees, one 6 feet tall and the other 8 feet tall, we can write this ratio three ways:

1) As a **fraction**:  $\frac{6 \text{ ft}}{8 \text{ ft}} = \frac{6}{8} = \frac{\cancel{2} \cdot 3}{\cancel{2} \cdot 4} = \frac{3}{4}$

2) With a **colon**:  $6 \text{ ft} : 8 \text{ ft} = 6 : 8 = 3 : 4$

3) With the word **to**:  $6 \text{ ft to } 8 \text{ ft.} = 6 \text{ to } 8 = 3 \text{ to } 4.$

Notice that in each case,

- The **order** of the numbers in a ratio is important! To write a ratio as a fraction, place the **first** number of the ratio in the **numerator**; place the **second** number in the **denominator**.
- The ratio is written in lowest terms.
- The units are not written as part of the ratio. Because a ratio compares two quantities with the **same** units – convert the units if they are different.

**Example 1:** Write each ratio as a fraction in lowest terms.

A) 15 pounds to 24 pounds

The units are the same, so we can write this ratio as

$$15 \text{ pounds to } 24 \text{ pounds} = \frac{15}{24} = \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 8} = \frac{5}{8}$$

B) 75 cents to \$1.25

Here, the units are not the same. Since \$1 = 100 cents, to convert \$1.25 to cents, drop the dollar sign and move the decimal point two places to the right.

$$\$1.25 = 1.25 = 125 \text{ cents}$$

$$75 \text{ cents to } \$1.25 = 75 \text{ cents to } 125 \text{ cents} = \frac{75}{125} = \frac{\cancel{3} \cdot \cancel{25}}{\cancel{5} \cdot \cancel{25}} = \frac{3}{5}$$

A **rate** is a comparison of two quantities with **different** units, such as 10 g per 180 mL. Like a ratio, a rate can be written as a fraction, with a colon, or with the word to. A rate is also expressed in lowest terms. Unlike a ratio, the units are written as part of the rate. For example, to write the rate “10 g per 180 mL” as a fraction in lowest terms, cancel the corresponding 0’s and keep the units:

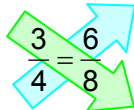
$$10 \text{ g per } 180 \text{ mL} = \frac{\cancel{10} \text{ g}}{\cancel{180} \text{ mL}} = \frac{1 \text{ g}}{18 \text{ mL}}$$

## PROPORTIONS

A **proportion** is a mathematical statement that two ratios or rates are equal. For example, whenever we write equivalent fractions, we create a proportion, such as the one shown below:

$$\frac{3}{4} = \frac{6}{8}$$

In a true proportion, the **cross products** are **equal**:



$$\frac{3}{4} = \frac{6}{8}$$

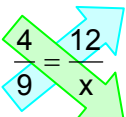
$$3(8) = 24$$

$$4(6) = 24$$

Because the cross products are equal, we can solve a proportion when one of the numbers is unknown.

**Example 2:** Solve  $\frac{4}{9} = \frac{12}{x}$

To solve the proportion 1) cross multiply the ratios, 2) write an equation; and 3) solve for the variable.



$$\frac{4}{9} = \frac{12}{x}$$

$$4 \cdot x = 9 \cdot 12$$

$$4x = 108$$

$$\frac{4x}{4} = \frac{108}{4}$$

$$x = 27$$

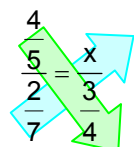
← Cross multiply the ratios

← Write an equation

← Solve the equation for x

**Example 3:** Solve  $\frac{5}{2} = \frac{x}{3}$

To solve the proportion, begin by finding the cross products:



$$\frac{5}{2} = \frac{x}{3}$$

$$\frac{2}{7} \cdot x = \frac{4}{5} \cdot \frac{3}{4}$$

← Cancel common factors

$$\frac{2}{7}x = \frac{3}{5}$$

$$\frac{7}{2} \cdot \frac{2}{7}x = \frac{3}{5} \cdot \frac{7}{2}$$

← Multiply both sides by the reciprocal of 2/7

$$x = \frac{21}{10}$$

## SOLVING PROBLEMS WITH PROPORTIONS

Proportions are often used to solve a variety of problems, such as estimating wildlife populations, scaling distances on a map, or calculating mixtures and dosages.

**Example 5:** Solve the problem by writing a proportion:

“The brewing directions on a bag of ground coffee recommend 2 tablespoons (tbs) of coffee for every 6 ounces (oz) of water. If the average-size coffee cup holds 6 ounces of coffee, how many tablespoons of coffee are needed to make eight 6-oz cups?”

To write the proportion,

- 1) **Identify the given ratio or rate and write it as a fraction.** In the problem above, we are given the rate “2 tablespoons of coffee for every 6 ounces of water.” We can write this rate as

$$2 \text{ tbs to } 6 \text{ oz} = \frac{2 \text{ tbs}}{6 \text{ oz}} = \frac{1 \text{ tbs}}{3 \text{ oz}}$$

- 2) **Assign a variable to represent the unknown quantity then write the second ratio or rate as a fraction.** Let  $x$  = the number of tablespoons needed to make eight 6-oz cups. Given that each coffee cup holds 6 ounces, we need 48 ounces of water to make eight 6-oz cups of coffee.. This gives us the rate:

$$x \text{ tbs to } 48 \text{ oz} = \frac{x \text{ tbs}}{48 \text{ oz}}$$

- 3) **Write the proportion.** Set the first ratio equal to the second. Note that, when we write a proportion, we are making a statement that the first ratio has the same value as the second. For this reason, the order in which we write a proportion is important! **Always place like units across from each other**, as shown:

$$\frac{1 \text{ tbs}}{3 \text{ oz}} = \frac{x \text{ tbs}}{48 \text{ oz}} \quad \leftarrow \text{tbs across from tbs} \quad \text{or} \quad \frac{3 \text{ oz}}{1 \text{ tbs}} = \frac{48 \text{ oz}}{x \text{ tbs}} \quad \leftarrow \text{oz across from oz}$$

Because the cross products are equal, inverting the ratios gives us an equivalent proportion.

- 4) **Solve the proportion.** To find the number of tablespoons, drop the units, cross multiply, and solve the resulting equation:

$$\begin{aligned} \frac{1}{3} &= \frac{x}{48} \\ 3x &= 48 \\ \frac{3x}{3} &= \frac{48}{3} \\ x &= 16 \text{ tbs} \end{aligned}$$

**Hint:** When you solve a problem with a proportion, you may find it helpful to state the problem in the following way: 2 tbs is to 6 oz as  $x$  tbs is to 48 oz.