SOLVING EQUATIONS

An equation is a mathematical statement that two expressions are equal. The solution of an equation is the value that when substituted for the variable makes the equation a true statement.

Our goal in solving an equation is to isolate the variable on one side of the equation and a number on the other side so the equation reads:

\[ \text{Variable} = \text{Number} \]

To achieve our goal, we use two principles of equality, the addition principle and the multiplication principle.

- Use the addition principle to move terms from one side of the equation to the other side. To move a term, add its opposite to both sides of the equation.

- Use the multiplication principle to solve for the variable. If the variable is multiplied by a number, divide both sides of the equation by that number. If the variable is divided by a number, multiply both sides of the equation by that number.

To solve equations, use the procedure outlined below.

Steps for Solving Equations

Step 1: Clear fractions and decimals by multiplying each term of the equation by the LCD (least common denominator).

Step 2: Remove the parentheses by distributing.

Step 3: Combine any like terms found on the same side.

Step 4: Use the addition principle to move the variable term to one side of the equation and the number to the other side.

Step 5: Multiply or divide to solve for the variable.

Step 6: Check the result in the original equation.

Example 1: Solve \(5(4t - 14) - 7 = 63\)

Solution: To solve the equation, begin by removing the parentheses.

\[
\begin{align*}
5(4t - 14) - 7 &= 63 & \text{distribute 5 to remove the parentheses} \\
20t - 70 - 7 &= 63 & \text{combine } -70 \text{ and } -7 \\
20t - 77 &= 63 & \\
20t &= 77 + 77 & \text{add 77 to both sides to isolate the variable term} \\
20t &= 140 & \text{add 140 to both sides to solve for the variable} \\
\frac{20t}{20} &= \frac{140}{20} & \\
t &= 7
\end{align*}
\]

Check:

\[
\begin{align*}
5(4(7) - 14) - 7 - 63 &? \\
5(28 - 14) - 7 &= 63 & ? \\
5(14) - 7 &= 63 & \\
70 - 7 &= 63
\end{align*}
\]
**Example 2:** Solve $5y - (2y + 7) = 4 - 5(2y - 3)$

**Solution:** Begin by removing the parentheses. Then simplify by combining like terms.

\[
5y - (2y + 7) = 4 - 5(2y - 3) \quad \text{← distribute the negative on the left and } -5 \text{ on the right}
\]

\[
5y - 2y - 7 = 4 - 10y + 15 \quad \text{← combine the like terms on the same side}
\]

\[
3y - 7 = 19 - 10y \quad \text{← add 10y to both sides to move the variable term to the left}
\]

\[
13y - 7 = 19 \quad \text{← add 7 to both sides to move the number to the right}
\]

\[
y = 2 \quad \text{← divide both sides by 13 to solve for the variable}
\]

**Example 3:** Solve: \( \frac{2}{3} x + 4 = \frac{1}{2} x - \frac{2}{3} \)

**Solution:** To clear the fractions, multiply each term by the LCD 6.

\[
\frac{2}{3} x + 4 = \frac{1}{2} x - \frac{2}{3}
\]

\[
\begin{align*}
2 \cdot \left( \frac{2}{3} x \right) + 4 \cdot 6 &= \frac{1}{2} \cdot x - \frac{2}{3} \cdot 6 \\
4x + 24 &= \frac{1}{2} x - 4
\end{align*}
\]

\[
4x + 24 = 3x - 4 \quad \text{← move the variable term to one side by adding } -3x \text{ to both sides}
\]

\[
x + 24 = -4 \quad \text{← move the number to the other side by adding } -24 \text{ to both sides}
\]

\[
x = -28
\]

**Example 4:** Solve $0.04n + 0.14(800 - n) = 0.4n$

**Solution:** To clear the decimals, multiply each term by the LCD 100.

\[
0.04n + 0.14 (800 - n) = 0.4n \quad \text{← move the decimal points 2 places to the right}
\]

\[
4n + 14 (800 - n) = 40n \quad \text{← distribute 14 to remove the parentheses}
\]

\[
4n + 11200 - 14n = 40n \quad \text{← combine 4n and } -14n
\]

\[
11200 - 10n = 40n \quad \text{← add 10n to both sides to isolate the variable term}
\]

\[
11200 = 50n \quad \text{← divide both sides by 50 to solve for the variable}
\]

\[
n = 224
\]