

## SOLVING EQUATIONS

An **equation** is a mathematical statement that two expressions are equal. The **solution** of an equation is the value that when substituted for the variable makes the equation a true statement.

Our **goal** in solving an equation is to isolate the variable on one side of the equation and a number on the other side so the equation reads:

$$\text{Variable} = \text{Number}$$

To achieve our goal, we use two principles of equality, the **addition principle** and the **multiplication principle**.

- Use the addition principle to move terms from one side of the equation to the other side. To move a term, add its opposite to both sides of the equation.
- Use the multiplication principle to solve for the variable. If the variable is multiplied by a number, divide both sides of the equation by that number. If the variable is divided by a number, multiply both sides of the equation by that number.

To solve equations, use the procedure outlined below.

### Steps for Solving Equations

**Step 1:** Clear fractions and decimals by multiplying each term of the equation by the LCD (least common denominator).

**Step 2:** Remove the parentheses by distributing.

**Step 3:** Combine any like terms found on the same side.

**Step 4:** Use the addition principle to move the variable term to one side of the equation and the number to the other side.

**Step 5:** Multiply or divide to solve for the variable.

**Step 6:** Check the result in the original equation.

**EXAMPLE 1:** Solve  $5(4t - 14) - 7 = 63$

Solution: To solve the equation, begin by removing the parentheses.

$$\begin{aligned} 5(4t - 14) - 7 &= 63 && \leftarrow \text{distribute 5 to remove the parentheses} \\ 20t - 70 - 7 &= 63 && \leftarrow \text{combine } -70 \text{ and } -7 \\ 20t - 77 &= 63 \\ +77 \quad +77 &&& \leftarrow \text{add 77 to both sides to isolate the variable term} \\ 20t &= 140 \\ \frac{20t}{20} &= \frac{140}{20} && \leftarrow \text{divide both sides by 20 to solve for the variable} \\ t &= 7 \end{aligned}$$

Check:

$$\begin{aligned} 5(4(7) - 14) - 7 &= 63 \\ 5(28 - 14) - 7 &= 63 \\ 5(14) - 7 &= 63 \\ 70 - 7 &= 63 \end{aligned}$$

**Example 2:** Solve  $5y - (2y + 7) = 4 - 5(2y - 3)$

**Solution:** Begin by removing the parentheses. Then simplify by combining like terms.

$$\begin{aligned}5y - (2y + 7) &= 4 - 5(2y - 3) && \leftarrow \text{distribute the negative on the left and } -5 \text{ on the right} \\5y - 2y - 7 &= 4 - 10y + 15 && \leftarrow \text{combine the like terms on the same side} \\3y - 7 &= 19 - 10y \\+10y & && +10y && \leftarrow \text{add } 10y \text{ to both sides to move the variable term to the left} \\13y - 7 &= 19 \\+7 & && +7 && \leftarrow \text{add } 7 \text{ to both sides to move the number to the right} \\13y &= 26 \\ \frac{13y}{13} &= \frac{26}{13} && \leftarrow \text{divide both sides by } 13 \text{ to solve for the variable} \\y &= 2\end{aligned}$$

**EXAMPLE 3:** Solve:  $\frac{2}{3}x + 4 = \frac{1}{2}x - \frac{2}{3}$

**Solution:** To clear the fractions, multiply each term by the LCD 6.

$$\begin{aligned}\frac{2}{3}x + 4 &= \frac{1}{2}x - \frac{2}{3} \\ \cancel{6} \cdot \left( \frac{\cancel{2}}{\cancel{3}_1} x \right) + \cancel{6} \cdot 4 &= \cancel{6} \cdot \left( \frac{\cancel{1}}{\cancel{2}_1} x \right) - \cancel{6} \cdot \left( \frac{\cancel{2}}{\cancel{3}_1} \right) && \leftarrow \text{multiply each term by the LCD } 6 \\4x + 24 &= 3x - 4 \\-3x & && -3x && \leftarrow \text{move the variable term to one side by adding } -3x \text{ to both sides} \\x + 24 &= -4 \\-24 & && -24 && \leftarrow \text{move the number to the other side by adding } -24 \text{ to both sides} \\x &= -28\end{aligned}$$

**EXAMPLE 4:** Solve  $0.04n + 0.14(800 - n) = 0.4n$

**Solution:** To clear the decimals, multiply each term by the LCD 100.

$$\begin{aligned}0.04n + 0.14(800 - n) &= 0.4n && \leftarrow \text{move the decimal points } 2 \text{ places to the right} \\4n + 14(800 - n) &= 40n && \leftarrow \text{distribute } 14 \text{ to remove the parentheses} \\4n + 11200 - 14n &= 40n && \leftarrow \text{combine } 4n \text{ and } -14n \\11200 - 10n &= 40n \\+10n & && +10n && \leftarrow \text{add } 10n \text{ to both sides to isolate the variable term} \\11200 &= 50n \\ \frac{11200}{50} &= \frac{50n}{50} && \leftarrow \text{divide both sides by } 50 \text{ to solve for the variable} \\224 &= n\end{aligned}$$